



# **Modelling welfare maximising mobile termination rates**

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# Executive summary

Frontier Economics has been asked to advise Vodafone Australia on the efficient mark-ups required to take into account:

- the presence of high fixed and common costs in the telecommunications market, which results in the need for a mark-up of price on incremental costs if a business is to be viable; and
- network externalities which mean that one person's decision to purchase a service (e.g. mobile subscription) may affect another person's welfare.

These two factors mean that the welfare maximising fixed-to-mobile (F2M) termination charge (also referred to as the Mobile Termination Rate or MTR) is not equal to the incremental cost of supply. The welfare maximising F2M termination charge must take into account the Ramsey pricing principles in allocating fixed and common costs among the various services supplied by a mobile operator (F2M, mobile subscription, and mobile-originated/outbound calls) and the network externalities generated by the use of these services.

## MODELLING APPROACH

The modelling approach we have adopted is based on the approach used to model welfare maximising F2M termination rates in the UK. The welfare modelling approach adopted here was subjected to considerable scrutiny in the UK by industry, regulators and expert advisers, and the debate has generated substantial insights about the application of Ramsey and externality principles to the assessment of the welfare impacts of the level of F2M termination charges.

We model demand functions for mobile subscription, mobile originated calls and F2M calls as three distinct services, with inter-related demand functions.

The model calculates a set of prices for each of the three services and the level of demand at the estimated prices that maximises social welfare for a given set of cost and relative price elasticity assumptions.

## MODEL INPUTS

To model the demand functions for mobile subscriptions, mobile-originated calls, and F2M calls we have gathered together information on prices, costs, demand and price elasticities for each of these services.

We have sought where possible to utilise numbers that appear to be supported by the ACCC's Final Decision. We have also drawn on other available empirical information and the outcomes of the recent review of mobile termination rates in the UK where the ACCC has not take a view on a particular parameter, and to provide credible ranges for use in the modelling.

In respect of LRIC and FCC costs, Vodafone has provided us with estimates prepared by PriceWaterhouse Coopers (PWC).<sup>1</sup>

Two scenarios are considered:

- Scenario 1: Fixed and Common Costs (FCC) include all network and central function costs; and
- Scenario 2: FCC include all network costs and non-network indirect costs.

Scenario 1 represents a lower bound on the estimate of FCC, while Scenario 2 provides an upper bound on the estimate of FCC.

At the request of Vodafone, our modelling has been undertaken on the basis that industry FCC are estimated by multiplying the estimates of Vodafone's FCC by four (based on the number of major mobile networks operating in Australia).

The non-cost and cost inputs used as the basis for the modelling are summarised in the tables below.

#### Non cost inputs

	<b>Initial (unscaled) average price</b>	<b>Demand (Annual)</b>	<b>Own Price Elasticity of Demand</b>	<b>Elasticity of volume to number of subscribers</b>
<b>Mobile subscriptions</b>	CIC	14.789 M	-0.3 to -0.6	0
<b>Mobile outbound calls</b>	CIC	15,472 M	-0.3 to -0.6	0.9 (0.7 by new subscribers + 0.2 by existing subscribers)
<b>F2M calls</b>	38.5 cpm	6,037 M	-0.3 to -0.6	0.4
<b>RG Factor</b>	1 to 1.5			

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<sup>1</sup> Frontier was not involved in the development of PWC's cost model. The PWC LRIC and FCC cost estimates used by Frontier differ from the cost estimates presented by PWC in its report in Annex 1 to Vodafone's undertaking. As outlined in that Annex 1, for the purposes of the cost modelling, PWC used a fully allocated cost approach to obtain estimates of the cost of each service which is analogous to the allocation of fixed and common costs on an equi-proportionate basis.

## Cost inputs

Cost estimates	Scenario 1	Scenario 2
Subscriptions (LRIC)	CIC	CIC
Mobile outbound (LRIC)	CIC	CIC
Fixed to mobile termination (Mobile LRIC)	CIC	CIC
Fixed to mobile calls mobile LRIC and fixed retention (includes mobile LRIC and 17.5 cpm fixed retention)	CIC	CIC
Fixed and Common Costs	CIC	CIC

## MODEL RESULTS AND CONCLUSIONS

The objective of the modelling is to determine the appropriate mark-up above the LRIC of F2M termination taking into account externality effects and the need to recover fixed and common costs using Ramsey pricing principles. As there is some discussion around the assumptions with respect to Ramsey pricing and externalities and the price elasticities of demand, we have undertaken three runs for each cost Scenario as follows:

- price elasticity of demand for F2M and mobile outbound calls of  $-0.6$ , and  $-0.3$  for subscriptions (based on the ACCC's preferred price elasticity assumptions for F2M calls and subscriptions);
- price elasticity of demand for F2M, mobile outbound calls and subscriptions of  $-0.3$  (based on parameters used in the analysis undertaken in the recent UK debate over mobile termination charges); and
- price elasticity of demand for F2M and mobile outbound calls of  $-0.3$  (based on the UK analysis) and  $-0.6$  for subscriptions (based on Optus' submissions to the ACCC).

In each case we have provided a breakdown of the total mark-up over the LRIC of F2M termination between the mark-up attributed to:

- the positive externalities associated with increased subscriptions; and
- the recovery of fixed and common costs according to Ramsey pricing principles.

The results are presented below for Scenarios 1 and 2. The mark-ups calculated are added to PWC's estimated LRICs of F2M termination of 12.46 cpm and 10.66 cpm for Scenarios 1 and 2 respectively to obtain the MTRs presented in the column to the right of the tables.

### Scenario 1: Model runs and MTRs

Run	Subscription Elasticity	Mobile outbound elasticity	F2M elasticity	Externality mark up (cpm)	Ramsey mark up (cpm)	Total mark up (cpm)	MTR (cpm)
1	-0.3	-0.6	-0.6	4.35	6.37	10.73	<b>23.19</b>
2	-0.3	-0.3	-0.3	6.30	8.53	14.83	<b>27.29</b>
3	-0.6	-0.3	-0.3	8.04	10.38	18.42	<b>30.88</b>

### Scenario 2: Model runs and MTRs

Run	Subscription Elasticity	Mobile outbound elasticity	F2M elasticity	Externality mark up (cpm)	Ramsey mark up (cpm)	Total mark up (cpm)	MTR (cpm)
1	-0.3	-0.6	-0.6	4.18	8.89	13.08	<b>23.74</b>
2	-0.3	-0.3	-0.3	6.20	12.19	18.39	<b>29.05</b>
3	-0.6	-0.3	-0.3	8.26	14.58	22.84	<b>33.50</b>

These results are as expected:

- moving from Scenario 1 to Scenario 2 the total mark ups are larger, given the larger FCC that must be allocated across different services in Scenario 2 relative to Scenario 1;
- in both Scenarios, moving through runs 1 to 2 and 2 to 3, F2M calls are increasingly less elastic relative to subscriptions meaning it is most efficient to increase the mark up on F2M calls to meet the fixed revenue requirement; and
- the externality effect is effectively constrained by our assumptions on the RG factor, meaning that the Ramsey effect becomes relatively more important in determining the mark-up for higher levels of FCC (between Scenarios 1 and 2).

This modelling demonstrates that mark ups on LRIC of between 10.73 and 22.84 cpm can be justified on the basis of the Ramsey and externality effects in the mobile telecommunications market. When added to the relevant LRICs this generates a range of MTRs from 23.19 to 33.50 cpm.



# 1 Introduction

Vodafone Australia has asked Frontier Economics to estimate the welfare maximising structure of prices for mobile termination services. Specifically, Frontier has been asked to consider the effect on F2M, mobile outbound, and mobile subscription services of adopting Ramsey pricing principles in allocating the fixed and common costs of a mobile network operator (MNO) and recognising network externalities.

This note outlines the results of our modelling. We discuss:

- the methodology and parameters used in the model (Section 2);
- the results obtained under a two Scenarios based on a range of assumptions (Section 3).

Prior to the discussion of the modelling approach, in Section 1.1 we briefly outline the economic principles underpinning the work.

## 1.1 WELFARE MAXIMISING PRICES

Given certain conditions (relating to such matters as the absence of externalities and certain types of cost functions) a set of competitive markets will produce an economically efficient, welfare maximising, allocation of resources. If these conditions hold, welfare will be maximised when the price of a good or service is set equal to its marginal cost of supply. That is, when the marginal revenue received from the sale of one additional unit equals the incremental costs of supplying that unit.

However, this result is only obtained under certain assumptions. Two of the key assumptions necessary to obtain this result are that:

- at the competitive equilibrium, marginal costs are equal to average costs, so the equation of prices with marginal costs is consistent with the condition of zero economic profits; and
- there are no externalities.

This arrangement is frequently violated in the real world. In the case of mobile telecommunications:

- the presence of high fixed and common costs in the provision of mobile origination and termination services results in marginal costs that are below average costs for the service as a whole. The presence of these fixed and common costs will mean that the setting of prices equal to marginal costs will not allow the recovery of the fixed and common costs; and
- network externalities mean that one person's decision to purchase a service (e.g. mobile subscription) may affect another person's welfare, hence price signals may need to be adjusted if private decision making is to lead to the maximisation of social welfare.

These two factors mean that the welfare maximising mobile termination charges will be different from the marginal cost of supply.

In a classic paper<sup>2</sup>, Ramsey showed that the rules for economically efficient prices would have to change to allow for the recovery of fixed or common costs if the setting of prices at marginal costs was inconsistent with the zero-profit constraint. In the last twenty or thirty years, it has been accepted among economists that the standard for economically-efficient prices in a multi-product firm is given by the Ramsey rules – and not by the rule that is suggested by the first fundamental theorem of welfare economics, that price should be equal to marginal cost.

The presence of externalities provides a further rationale for moving prices away from marginal costs. If one person's consumption of a service (e.g. mobile subscription) generates a positive externality (that is, a benefit to others that the subscriber can not privately capture), the welfare maximising price for the subscription service may be lower than it would be in the absence of such an externality.

These (standard) principles for the regulation of prices based on marginal costs (with adjustments to allow for the optimal recovery of fixed and common costs and to allow for externalities) have been challenged for two principal reasons. Neither of these challenges have questioned any of the theory that is summarised above. Both challenges are based on the practical problems that confront regulators in setting prices.

The first challenge has come from regulators who are concerned about the reliability of the elasticity estimates that are needed if Ramsey prices are to be calculated. If marginal costs are very low and fixed and common costs are very high, then any setting Ramsey prices based on marginal costs will be highly dependent on the elasticity estimates. If these elasticity estimates are subject to a great deal of uncertainty, then the Ramsey prices will also be subject to a great deal of uncertainty.

This has led many regulators to base regulated prices on average long-run incremental cost rather than marginal cost. Average long-run incremental cost accounts for all the costs that are forgone as the result of a decision to include the service in question among the range of services offered by the market. So these long-run incremental costs include a lot more of the costs than do the marginal costs. Because of this, the mark-up that is needed to recover any fixed and common costs is smaller than is needed if regulated prices are based on marginal costs.

The second challenge has come from the literature on cross-subsidies and contestability. This literature (in particular, the writing of Faulhaber and Baumol) has argued that average long-run incremental cost has certain properties that make it preferable to marginal cost as a base for determining regulated prices.<sup>3</sup> In particular, LRIC is the minimum price below which one may suspect that a price may be predatory.

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<sup>2</sup> Ramsey, F P (1927), "A Contribution to the Theory of Taxation", *Economic Journal*, vol 37, pp. 47-61.

<sup>3</sup> See William J Baumol and J Gregory Sidak, *Toward Competition in Local Telephony*, MIT Press, 1994, pp. 65 ff.

As a result of these challenges, regulated prices are based on average long-run incremental costs rather than marginal costs. This does not mean that regulators should set prices equal to LRIC. Viable suppliers need to be able to recover their fixed and common costs; and this requires a mark-up on LRIC. Furthermore, prices must also allow for externalities.

The welfare maximising charge for mobile termination services must therefore:

- provide for a mark-up on LRIC according to the Ramsey pricing principles to allow for the recovery of fixed and common costs; and
- adjust these mark-ups so as to take account of external effects.

Our approach to estimating welfare maximising termination charges is discussed in Section 2.

## 2 Modelling approach

The modelling approach we have adopted is based on the approach used to model welfare maximising F2M termination charges in the UK. We model demand functions for mobile subscription, mobile originated calls and fixed to mobile calls as three distinct services, with inter-related demand functions.

The model calculates a set of charges for each of the three services and the level of demand at the estimated prices that maximises social welfare (by maximising consumer surplus) for a given set of cost and relative price elasticity assumptions. These prices are then compared with the LRIC to determine the welfare maximising mark-up. The approach used in the model is outlined diagrammatically in Annex 1 and mathematically in Annex 2.

In undertaking the modelling exercise we have sought to use parameters specific to Australia and in particular, those that appear to be supported by the ACCC in its Final Decision. Where these are not readily available, or where there is a degree of uncertainty about the estimates, we have also taken into account evidence from other countries and the regulatory debate on this matter in the UK, in particular, the conclusions drawn by Oftel (and its successor Ofcom) and the Competition Commission. Wherever we have drawn on the approach adopted in the UK we also present model outputs to demonstrate the impact of those parameters on the results.

### 2.1 WELFARE MODELLING IN THE UK

There has been substantial empirical work done over the course of the regulatory inquiries in the UK specifically addressing the issue of the welfare maximising mobile termination rate.

There was contention between the various models developed to ascertain the welfare optimising mark-up on the Long Run Incremental Cost (LRIC) of F2M termination. Models were prepared for the regulator Oftel (by Dr Rohlf's) and by those providing mobile terminations services. Models included those developed by Frontier Economics UK (for Vodafone UK), DotEcon and Lexecon.

There was a long process of vigorous discussion as to the appropriateness of the assumptions used in each of the models and on the differing model outputs. Of note, the debate surrounding the welfare modelling was not one of whether or not network externalities are relevant but how they are characterised and best incorporated into the models.

The key issues of debate included:

- the extent to which MNOs internalised the externalities in their pricing structures;
- the magnitudes of the various own and cross price elasticities, the magnitudes of the various volume elasticities with respect to the number of mobile subscribers; and

### Modelling approach

- the role and appropriate level of the Rohlfs-Griffin factor (which reflects the relative public and private benefits of additional mobile subscriptions).

The results of the Frontier Economics model have been used in submissions to Ofcom, and the Competition Commission in the UK for determining the welfare maximising F2M termination rate or more specifically, the optimal mark-up above costs of mobile termination. They have also been reviewed by Professor James Mirrlees in his expert witness statement to the UK High Court submitted as part of the evidence presented by Vodafone UK in their appeal of the Competition Commission's verdict.

We consider the inputs to and outcomes of the UK process can provide some valuable guidance for the modelling of welfare maximising mark-ups in Australia given:

- the high level of detailed analysis such as econometric assessment of different price elasticities in the UK market;
- the extensive peer review over at least four years; and
- the fact that some level of consensus was reached between competing interests.

## 2.2 PARAMETERS USED IN THE MODEL

The demand system of the Frontier Model consists of three mobile services (mobile subscriptions, mobile-originated/outbound calls, and F2M calls). We note that the demand for subscriptions is “derived demand” because customers do not receive value from subscriptions directly, but rather from the ability a subscription affords them to receive and make calls.<sup>4</sup> For each service the inputs required are:

- 1) its average price;
- 2) its cost (long run incremental cost, or LRIC);
- 3) its annual demand (quantities) for the whole market;
- 4) an estimate of the costs incurred by all mobile operators that are fixed and common across all the services that they offer.

The model also requires elasticity estimates, in particular the own price elasticities of each of the three services measured at current levels of demand and the cross-price elasticities between the services.<sup>5</sup> In addition, the elasticities of the volume of F2M and mobile outbound calls with respect to the number of mobile subscriptions must be considered based on expectations about the private and public benefits of new subscriptions.

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<sup>4</sup> We note that our approach of modelling three demand curves is theoretically valid, and in practice theoretically equivalent to modelling at two part tariff. The two services – connection and consumption are complements (Laffont and Tirole, 2000, p. 69).

<sup>5</sup> The model is based on linear demand schedules, hence the own-price elasticities will vary depending on the prices applying in the model.

## 2.3 OWN PRICE ELASTICITIES OF DEMAND

In this section we outline the rationale for the range of own price elasticities used to determine the results presented in Section 3.

### 2.3.1 Fixed-to-mobile calls

The ACCC has maintained a view throughout the review that  $-0.6$  is an appropriate own price elasticity of demand for F2M calls.<sup>6</sup> The ACCC states that it has ‘considered a variety of sources of information about the magnitude of the own price elasticity – market inquiries; estimates used by market analysts; and the only recent formal professional study [by DotEcon]’ and that ‘All of these sources suggest an elasticity in the range between roughly  $-0.4$  and  $-0.8$ , with a mid-value of  $-0.6$ ’ and that an elasticity within this range is supported by its own (undetailed) market enquiries.<sup>7</sup>

We note that DotEcon used the lower price elasticity from its range ( $-0.4$ ) in its welfare modelling in the UK, and that after much debate in that market there was general agreement to the use of  $-0.3$  based on statistical studies that indicated the price elasticity of demand may have been as low as  $-0.178$ . Optus has argued during this review that a price elasticity of demand of  $-0.1$  may be appropriate, although this has been criticised by the ACCC as being an underestimate.

Given the divergence of available information, and an absence of any statistical analysis of the actual elasticity in the Australian market, we have utilised a range from  $-0.6$  to  $-0.3$  in our modelling.

### 2.3.2 Mobile outbound calls

The ACCC does not appear to endorse a particular price elasticity of demand for mobile outbound calls. We have therefore drawn on other available information:

- Frontier Economics’ (Australia) review for Vodafone New Zealand found that elasticities for mobile originated calls ranged from  $-0.09$  to  $-0.80$ <sup>8</sup>; while
- agreement was reached in the UK to the use of  $-0.3$  (the same as for F2M) although some modelling by Frontier Economics UK was undertaken on the basis that mobile outbound call price elasticity of demand would be higher than the F2M price elasticity due to the feedback effect. This is because raising of prices also causes subscribers to leave the network, which further impacts on mobile outbound calls even in the absence of any externalities.

Given the divergence of available information, and an absence of any statistical analysis of the actual elasticity in the Australian market, we follow the precedent used in the UK of adopting similar own price elasticities of demand for mobile outbound and F2M calls, and adjusting for the feedback effect.

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<sup>6</sup> ACCC, Final Decision, p. 154.

<sup>7</sup> Ibid, note 409, p. 154

<sup>8</sup> Note that one study was excluded as a potential outlier – Ahn and Lee (1999) estimated a price elasticity of demand of  $-30.62$  for mobile originated calls.

We have therefore used a range of  $-0.3$  to  $-0.6$  for the mobile outbound price elasticity of demand in developing the results provided in Section 3.

### 2.3.3 Mobile subscriptions

The ACCC does not appear to endorse a particular price elasticity of demand for mobile subscription, but points to studies that range between  $-0.55$  and  $-0.3$  when considering Optus' submission of  $-0.6$  and  $-0.54$  (and  $-1.0$  initially proposed by Optus).<sup>9</sup>

Given the divergence of available information, and an absence of any statistical analysis of the actual elasticity in the Australian market, we have utilised a range from  $-0.6$  to  $-0.3$  in our modelling.

## 2.4 VOLUME ELASTICITIES

Volume elasticities measure the change in the number of calls associated with a change in the number of mobile subscribers.

At its simplest, the elasticity of F2M calls with respect to the number of mobile subscribers is a pure externality effect. All these calls, generated by the existence of mobile subscribers, are made by people other than the subscribers themselves. The fact that these third parties make calls indicates that they must get value from making them. Hence an externality is directly demonstrated. The magnitude of this externality can be computed using these quantity-on-quantity effects and the own-price elasticity of mobile subscription.

The ACCC has taken the position that it has received no evidence to show the size of such externalities in the mobile communications market.<sup>10</sup> Furthermore, the ACCC does not consider that network externality arguments should be taken into account in determining the welfare maximising allocation of fixed and common costs to mobile termination.<sup>11</sup>

The elasticity of demand for the various services with respect to the *number* of mobile subscribers is more difficult to ascertain from the literature than for own price elasticities. However, it is possible to obtain direct empirical evidence of the size of externalities from parameters relating to the elasticity of demand for the various services with respect to the number of mobile subscribers. The approach we adopt for F2M and mobile outbound is outlined in sections 2.4.1 and 2.4.2 respectively.

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<sup>9</sup> ACCC, Final Decision, p. 161.

<sup>10</sup> Ibid, p. 168.

<sup>11</sup> Ibid, pp. 159, 168-9

### 2.4.1 Change in the volume of F2M calls from a change in the number of subscribers

Frontier Economics (UK) estimated from their econometric studies in the UK that the change in the volume of F2M calls for an increased subscriber was 0.4.<sup>12</sup> This can be interpreted as entirely an externality effect as it reflects the number of new F2M calls that are made as the result of a new subscription. DotEcon estimated this parameter to be 0.6. Both estimates were derived using time series econometrics.

In the UK it was argued that these parameters might be too high because they included a general trend for the increasing use of telephones which was not driven by the number of mobile subscribers. However, both Frontier and DotEcon took great efforts to control for taste trends in their econometric analysis.

In the absence of empirical data for Australia, we have used the bottom end of these estimates of 0.4 as a starting point. In effect, when the RG factor is calibrated to 1.5 (discussed below) this volume elasticity is constrained to a maximum of 0.11 (well below 0.4).

### 2.4.2 Change in the volume of mobile outbound calls from a change in the number of subscribers

The elasticity of mobile outbound calls with respect to the number of mobile subscribers will encompass both an externality effect (the number of calls generated by parties other than the new subscribers) and the calls made by the new subscriber.

Frontier Economics' (UK) econometric analysis found that in the UK the change in the volume of mobile outbound calls for an increase of one subscriber was 0.9. Of this, around 0.7 was the private effect of new subscribers making calls and around 0.2 is the externality effect of people making mobile-to-mobile (M2M) calls to the new subscribers.<sup>13</sup> This is consistent with findings in the UK by DotEcon and others that the ratio of marginal mobile use to average mobile use was less than one.

In the absence of empirical data for Australia, we have used the estimate of 0.9, of which 0.7 is a private effect, and 0.2 an externality effect as a starting point. In effect, when the RG factor is calibrated to 1.5 (discussed below) the volume elasticity related to the external effect is constrained to a maximum of 0.055 (well below 0.2).

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<sup>12</sup> 'Econometric analysis of price elasticities of demand for mobile services', Frontier Economics, March 2002.

<sup>13</sup> Ibid.



### 2.4.3 Change in the volume of subscriptions from a change in the number of subscribers

While at certain stages in the expansion of the mobile market one might argue that the more subscribers there are the more people would want to become subscribers. However this effect is neither uniform nor continuous, but necessarily becomes exhausted (tends to zero) as mobile penetration increases towards saturation. We believe that the key externality effects of concern are captured by the other two quantity-on-quantity effects and so we set this elasticity at 0 in our modelling.

### 2.4.4 The ratio of private to public benefits

In assessing the potential impact of the above externality effects on the appropriate mark up, it is necessary to consider the RG factor. The RG factor is defined as “the ratio of marginal social benefit to marginal private benefit of an additional subscriber on the mobile network.”

The RG factor should be determined empirically based on the relevant volume elasticities. This was an area of considerable debate in the UK, given the RG factor is very sensitive to the values of the volume elasticities described above.

Given we do not have any direct measurements of these factors for Australia, we believe that in the circumstances it is appropriate to constrain the values of the volume elasticities in our modelling to ensure an acceptable value for the RG factor, measured at the relevant prices and quantities for Australia.

In the modelling in which we account for network externalities, we use an RG factor of 1.5, which is the mid point of the range chosen by the UK Competition Commission and significantly lower than that found by Frontier Economics (UK) in the UK analysis. This means that the volume elasticities attributed to the externality effects of new subscribers (0.2 for new mobile calls per subscriber and 0.4 for new F2M calls per subscriber) are adjusted downwards. Hence we believe it to be a conservative assumption where we assess the externality effect on the appropriate level of mark-up.

For the modelling where we do not account for the externality effect, we set the RG factor to 1, implying there is no external benefit above the private benefit from additional subscribers in these runs.

## 2.5 COST INFORMATION

Cost information has been provided by Vodafone Australia based on its own internal analysis carried out by PriceWaterhouse Coopers (PWC). We note that the PWC LRIC and FCC cost estimates used by Frontier differ from the cost estimates presented by PWC in its report in Annex 1 to Vodafone’s undertaking.<sup>14</sup>

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<sup>14</sup> Frontier was not involved in the development of PWC’s cost model. The PWC LRIC and FCC cost estimates used by Frontier differ from the cost estimates presented by PWC in its report in Annex 1 to Vodafone’s undertaking. As outlined in that Annex 1, for the purposes of the cost modelling,

A key issue is the appropriate allocation of costs between FCC and the LRIC of services. Two cost scenarios are used in our modelling:

- Scenario 1: FCC include all network and central function costs; and
- Scenario 2: FCC include all network costs and non-network indirect costs.

Scenario 1 represents a lower bound on the estimate of FCC, while Scenario 2 provides an upper bound on the estimate of FCC.

### 2.5.1 Fixed and common costs

PWC's estimates for Vodafone's FCC are outlined in Table 2.1. To determine industry-wide FCC Vodafone has requested that we scale these costs up by the number of major mobile carriers/networks – i.e. multiplying PWC's estimate of Vodafone's FCC by four. This is based on the presence of four major mobile networks operating in Australia: Telstra-GSM; Telstra-CDMA; Optus; Vodafone). The industry-wide estimates of FCC are provided in Table 2.1.

Table 2.1: Fixed and Common Costs

	<b>Scenario 1 (\$M)</b>	<b>Scenario 2 (\$M)</b>
<b>PWC estimate of Vodafone FCC</b>	CIC	CIC
<b>Industry-wide estimate of FCC</b>	CIC	CIC

We consider that the industry-wide estimates of FCC are likely to be conservative figures because:

- Vodafone's advice is that network FCC are driven by geographic coverage. Telstra has a greater geographical coverage than the other carriers. This is especially so in the case of its CDMA network; and
- no allowance is made for Hutchison's FCC of supporting its networks.

### 2.5.2 Long run incremental costs

PWC has provided us with estimates of the LRIC for each of the three services – subscription, mobile outbound, and fixed to mobile calls. The LRIC of mobile outbound calls used is the average end-to-end cost of three types of calls: on-net (own mobile network) calls; off-net calls to fixed network; off-net calls to other mobile networks. These are weighted by their respective volumes (minutes) and include the relevant costs of origination and termination.

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PWC used a fully allocated cost approach to obtain estimates of the cost of each service which is analogous to the allocation of fixed and common costs on an equi-proportionate basis.

The LRIC figures provided by PWC for both scenarios are provided in Table 2.2.

Table 2.2: Long run incremental costs

LRIC	Scenario 1	Scenario 2
<b>Subscriptions</b>	CIC	CIC
<b>Mobile outbound</b>	CIC	CIC
<b>Fixed to mobile termination (Mobile LRIC)</b>	CIC	CIC
<b>Fixed to mobile calls mobile LRIC and fixed retention (includes mobile LRIC and 17.5 cpm fixed retention<sup>15</sup>)</b>	CIC	CIC

## 2.6 PRICE AND DEMAND INPUTS

### 2.6.1 Prices

We use the following price data:

- mobile subscriptions are priced at CIC each on average. This is calculated from Vodafone data on subscription revenues divided by the number of subscribers. This compares with the Optus estimate of \$145 a year;<sup>16</sup>
- mobile outbound call prices are estimated to be CIC cpm based on Vodafone Australia mobile revenues divided by the number of internally sourced minutes;<sup>17</sup> and
- F2M call average prices are estimated at 38.5 cpm as used by the ACCC in its draft and final decisions.<sup>18</sup>

### 2.6.2 Demand

We use the following demand information:

- mobile subscriptions annual demand of 14.789 M, based on the reported number of subscribers for all carriers in the third quarter of 2003;<sup>19</sup>

<sup>15</sup> This is referred to in the UK as the fixed operator's 'retention'. In Australia, the fixed retention is determined by subtracting the MTR (21 cpm) from the FTM retail price (38.5 cpm).

<sup>16</sup> Optus, 'Optus Submission to Australian Competition and Consumer Commission on Mobile services (Submission 2, June 2003), p. 31, para 4.24. Final Decision, p. 160, note 416.

<sup>17</sup> We understand that this is probably a conservative basis for modelling the industry to the extent that Vodafone's average prices may be higher than those of the other carriers.

<sup>18</sup> ACCC Draft Decision, p. 126. Final Decision, p. 153.

<sup>19</sup> Merrill Lynch, 'Global Wireless Matrix 3Q03', p. 40. This compares with the number used Optus of 13.9 million subscribers: Optus Submission 2 (June 2003), 31; ACCC Final Decision, p. 160, note 416.

## Modelling approach

- mobile outbound calls of 15,472 M minutes, based on information on average monthly minutes of use (MOU) per user on each network from quarter four 2002 to quarter three 2003, grossed up for twelve months, multiplied by the number of subscribers per network and then halved to reflect that MOUs measured both incoming and outgoing calls;<sup>20</sup> and
- F2M calls of 6,037 M minutes a year as considered appropriate by the ACCC.<sup>21</sup>

### 2.6.3 Consideration of total revenues and costs

We determined total revenues and costs across the mobile industry based on:

- the quantities of mobile outbound and F2M calls described above;
- the prices for subscriptions and mobile outbound calls outlined above, and the MTR of 21 cpm for FTM calls; and
- the LRICs provided in Table 2.2 (excluding the fixed line retention for FTM calls).

We found that revenues were indeed very close to costs (with the gap between revenues and costs of minus 2 percent of revenues under Scenario 1 and plus 2.3 percent of revenues under Scenario 2).

This provides an appropriate starting point for the model, which solves for prices and quantities to maximise the total welfare subject to a zero total profit constraint (across the three services – subscription, mobile outbound and F2M).

We note there may be a gap between total costs and revenues driven by a number of factors, including:

- the conservative nature of the estimate of FCC used given likely differences across networks with different geographic coverage and no allowance for Hutchison's costs;
- that the prices used in the modelling may not in fact reflect actual average prices across the market; and
- the investment cycle: revenues may exceed costs in a particular year especially as the market nears the end of the 2G investment cycle. This may be necessary to generate a return on the costs incurred in the past to establish mobile telecommunications networks.

To the extent that costs exceed revenues, all else being equal, we would expect a higher mark up would be required to meet the zero profit constraint.

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<sup>20</sup> Based on data provided in Merrill Lynch, 'Global Wireless Matrix 3Q03', p. 40. The Merrill Lynch data shows that Vodafone has 14 per cent of the mobile outbound market which, when used to extrapolate Vodafone's figure of 2,138,614,352 outbound minutes to a national figure, provides an estimate of 15,267 which compares favourably with the above estimate.

<sup>21</sup> ACCC, Final Decision, p. 153.

## 2.7 SUMMARY OF INPUTS

The inputs used in the modelling are summarised in the tables below.

Table 2.3: Summary of non cost inputs

	Initial (unscaled) average price(\$)	Demand (Annual)	Own Price Elasticity of Demand	Elasticity of volume to number of subscribers
<b>Mobile subscriptions</b>	CIC	14.789 M	-0.3 to -0.6	0
<b>Mobile outbound calls</b>	CIC	15,472 M	-0.3 to -0.6	0.9 (0.7 new subscribers + 0.2* existing subscribers)
<b>F2M calls</b>	38.5 cpm	6,037 M	-0.3 to -0.6	0.4*
<b>RG Factor</b>	1 to 1.5			

\* Note that these volume elasticities were scaled downwards when the RG factor was calibrated to 1.5, and are effectively set to 0 where the RG factor is set to 1.

Table 2.4: Summary of cost inputs

Cost estimates	Scenario 1	Scenario 2
<b>Subscriptions (LRIC)</b>	CIC	CIC
<b>Mobile outbound (LRIC)</b>	CIC	CIC
<b>Fixed to mobile termination (Mobile LRIC)</b>	CIC	CIC
<b>Fixed to mobile calls mobile LRIC and fixed retention (includes mobile LRIC and 17.5 cpm fixed retention<sup>22</sup>)</b>	CIC	CIC
<b>Fixed and Common Costs</b>	CIC	CIC

<sup>22</sup> This is referred to in the UK as the fixed operator's 'retention'. In Australia, the fixed retention is determined by subtracting the MTR (21 cpm) from the FTM retail price (38.5 cpm).

### 3 Results

In this section we discuss the results of the welfare modelling. The objective of the modelling is to determine the appropriate mark-up of above the LRIC of F2M termination taking into account externality effects and the need to recover fixed and common costs using Ramsey pricing principles. As there is some discussion around the assumptions with respect to Ramsey pricing and externalities, and the appropriate level of price elasticity of demand we have undertaken a number of runs for Scenarios 1 and 2 as follows:

- price elasticity of demand for F2M and mobile outbound calls of  $-0.6$ , and  $-0.3$  for subscriptions (based on the ACCC's preferred price elasticity assumptions for F2M calls and subscriptions);
- price elasticity of demand for F2M, mobile outbound and subscriptions of  $-0.3$  (based on parameters used in the analysis undertaken in the recent UK debate over mobile termination charges); and
- price elasticity of demand for F2M and mobile outbound calls of  $-0.3$  (based on the UK analysis) and  $-0.6$  for subscriptions (based on Optus' submissions to the ACCC).

In each case we have provided a breakdown of the total required mark-up over the LRIC of F2M termination between the mark-up attributed to:

- the positive externalities associated with increased subscriptions; and
- the recovery of fixed and common costs according to Ramsey pricing principles.

In each case we have provided a breakdown of the total mark-up over the LRIC of F2M termination between the mark-up attributed to:

- the positive externalities associated with increased subscriptions; and
- the recovery of fixed and common costs according to Ramsey pricing principles.

The results are presented below for Scenarios 1 and 2. The mark-ups calculated are added to PWC's estimated LRICs of F2M termination of 12.46 cpm and 10.66 cpm for Scenarios 1 and 2 respectively to obtain the MTRs presented in the column to the right of the tables.

Table 3.1: **Scenario 1**: Model runs and MTRs

Run	Subscription Elasticity	Mobile outbound elasticity	F2M elasticity	Externality mark up (cpm)	Ramsey mark up (cpm)	Total mark up (cpm)	MTR (cpm)
<b>1</b>	-0.3	-0.6	-0.6	4.35	6.37	10.73	<b>23.19</b>
<b>2</b>	-0.3	-0.3	-0.3	6.30	8.53	14.83	<b>27.29</b>
<b>3</b>	-0.6	-0.3	-0.3	8.04	10.38	18.42	<b>30.88</b>

Table 3.2: **Scenario 2**: Model runs and MTRs

Run	Subscription Elasticity	Mobile outbound elasticity	F2M elasticity	Externality mark up (cpm)	Ramsey mark up (cpm)	Total mark up (cpm)	MTR (cpm)
<b>1</b>	-0.3	-0.6	-0.6	4.18	8.89	13.08	<b>23.74</b>
<b>2</b>	-0.3	-0.3	-0.3	6.20	12.19	18.39	<b>29.05</b>
<b>3</b>	-0.6	-0.3	-0.3	8.26	14.58	22.84	<b>33.50</b>

These results are as expected:

- moving from Scenario 1 to Scenario 2 the total mark ups are larger, given the larger FCC that must be allocated across different services in Scenario 2 relative to Scenario 1;
- in both Scenarios, moving through runs 1 to 2 and 2 to 3, F2M calls are increasingly less elastic relative to subscriptions meaning it is most efficient to increase the mark up on F2M calls to meet the fixed revenue requirement; and
- the externality effect is effectively constrained by our assumptions on the RG factor, meaning that the Ramsey effect becomes relatively more important in determining the mark-up for higher levels of FCC (between Scenarios 1 and 2).

### 3.1 CONCLUSIONS

This modelling demonstrates that mark ups on LRIC of between 10.73 and 22.84 cpm can be justified on the basis of the Ramsey and externality effects in the mobile telecommunications market. When added to the relevant LRICs this generates a range of MTRs from 23.19 to 33.50 cpm.

## Results

## Annex 1: Diagrammatic representation of the modelling methodology

The model calculates optimal prices by selecting the set of prices that maximise total welfare subject to the constraint that the operator should make zero profits overall. Society's welfare is defined as the sum of the areas under the three demand curves and above the cost lines for the three services. This is shown, for the case of mobile subscription, by the shaded area in Figure A1.1,<sup>23</sup> where  $c_s$  is the cost of subscription.

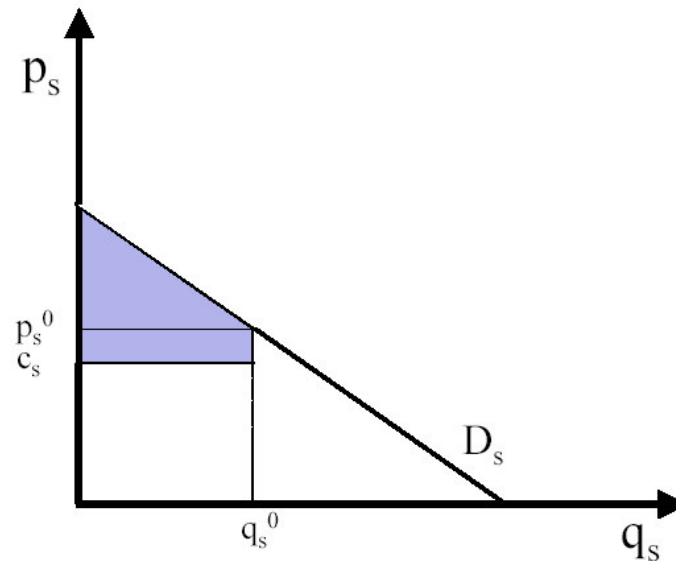


Figure A1.1: Total welfare

In the absence of both fixed and common costs and network externalities, total welfare is maximised when prices in each of the three markets are set equal to marginal cost. If there are fixed and common costs, prices set equal to average LRIC would generate negative profits. The model would therefore adjust prices in a way that would ensure zero profits, while minimising the welfare-loss when prices deviate from LRIC. This is similar to the Ramsey pricing result.

In the presence of a network externality, the welfare loss associated with a deviation in the price of mobile subscription from the LRIC of subscription may be compensated for by an increase in the welfare associated with an increase in the volume of mobile outgoing and fixed-to-mobile calls. In this situation, it may be welfare-maximising to set the price of mobile subscription below the LRIC and raise the prices of other services above their respective LRIC. The optimal

<sup>23</sup> This definition of total welfare is the same as the sum of consumer surplus and producer surplus. In all the models that we have considered, welfare is optimised subject to a zero-profit constraint. The producer surplus is therefore equal to zero.



set of prices is determined by the point at which the marginal loss of welfare caused by prices diverging further from LRIC equals the marginal gain in welfare associated with the greater volumes of incoming calls caused by the additional mobile subscribers.

This is illustrated in Figure A1.2 for the case of two demand functions – subscription and fixed-to-mobile calls. A similar process applies to the demand for mobile-to-mobile calls.

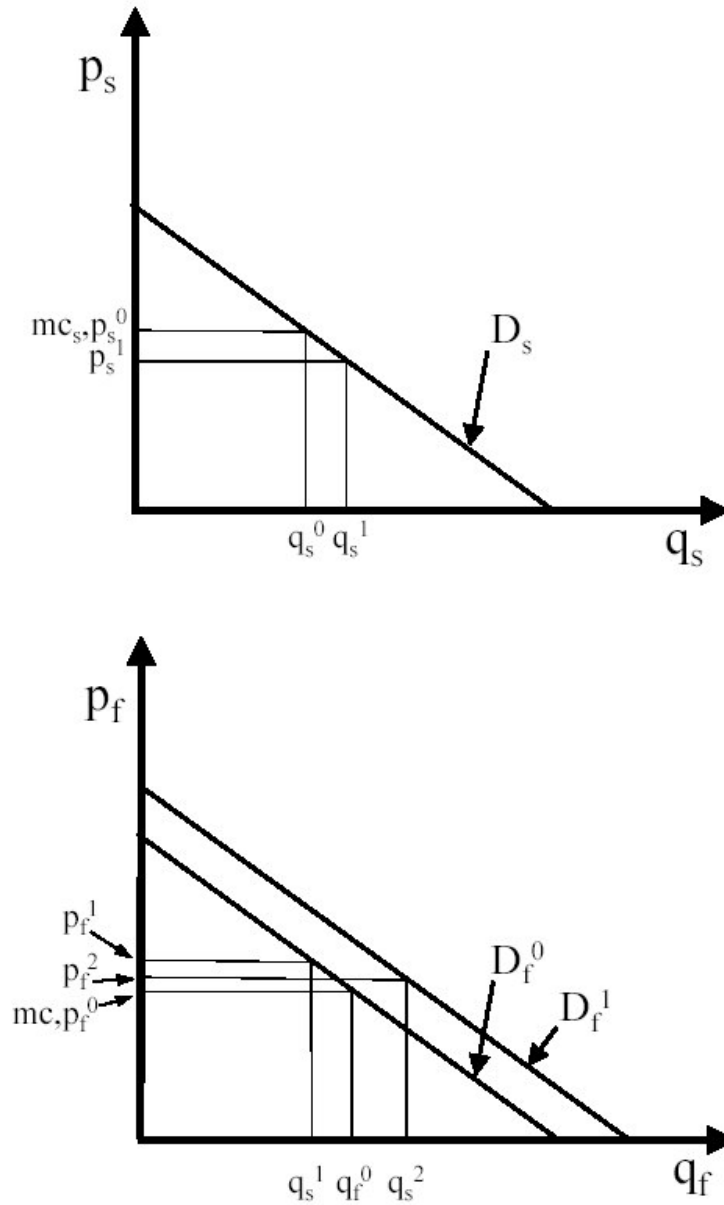


Figure A1.2: Effect of the network externality on fixed-to-mobile calls

If the price of mobile subscription were to fall below costs, from  $p_s^0$  to  $p_s^1$ , the number of subscribers would increase from  $q_s^0$  to  $q_s^1$  and the operator will make

## Annex 1: Diagrammatic representation of the modelling methodology

a loss on subscription. Since the operator is required to break-even, the price of fixed-to-mobile calls must rise. In the absence of a network externality, the price of fixed-to-mobile calls would rise above costs from  $p_f^0$  to  $p_f^1$ . This would result in a reduction in the quantity of fixed-to-mobile calls from  $q_f^0$  to  $q_f^1$  and a corresponding loss of welfare for fixed-to-mobile callers. However, in the presence of a network externality, the rise in the number of subscribers increases the number of fixed-to-mobile calls. This is shown by the shift in the demand curve from  $D_f^0$  to  $D_f^1$ . Instead of there being a loss in welfare generated by fewer fixed-to-mobile calls when the price of fixed-to-mobile calls was raised from  $p_f^0$  to  $p_f^1$ , there is an increase the welfare of fixed-to-mobile callers arising from the increase in the volume of fixed-to-mobile calls. The price required for the operator to break-even at a subscription price of  $p_s^1$  is therefore  $p_f^2$  which is lower than  $p_f^1$ . The optimal price of fixed-to-mobile calls is the point at which the welfare loss of a further drop in the price of mobile subscription equals the welfare gain associated with an increase in the volume of fixed-to-mobile calls. A similar analysis applies to mobile-to-mobile calls.

## Annex 1: Diagrammatic representation of the modelling methodology

## Annex 2: Mathematical basis for modelling optimal call termination charges

In the Frontier (UK) model, the outgoing calls service is summarised by two separate demand functions – demand for subscriptions and demand for mobile outgoing calls. This was chosen as an alternative to modelling subscription charges by way of a single demand function that incorporates two separate charges – a fixed charge per subscriber and a variable charge that is related to the number of calls (i.e. a two-part tariff model). The reasons for this approach are as follows:

1. In practice, it is difficult to apply a two-part tariff model to calculate the actual level of optimal prices, because the formula includes terms that are not directly quantifiable from empirical data. In particular, the willingness of consumers to pay a subscription charge depends on the distribution of preferences for mobile services within the population, which is not directly observable.
2. The approach of modelling three demand curves is both theoretically valid and, in practice, theoretically equivalent to modelling a two-part tariff. Furthermore our approach is consistent with previous studies conducted in the context of the present investigation. [in the UK]<sup>24</sup>

The following section derives a formula for optimal prices in a two-part tariff model, based on a standard model of two-part tariffs. As noted above the model used is theoretically equivalent to this.

### ANNEX 2.1 OPTIMAL PRICES WITH TWO-PART TARIFFS

Assume that there are two groups of customers – fixed customers and mobile customers.

#### *Fixed customers*

The welfare of fixed subscribers from fixed-mobile calls is a function of the price of fixed-mobile calls,  $p_f$ , and the number of mobile subscribers,  $N$ . If it is assumed that the welfare of fixed-mobile callers is a linear function of the number of mobile subscribers, total welfare of fixed-mobile callers,  $V_f$ , can be written as:

$$V_f(N, p_f) = Nv(p_f)$$

$v(p_f)$  denotes the aggregate benefit derived by all fixed line customers from the addition of a typical subscriber to the mobile network and is a direct measure of the externality. In keeping with the literature, we ignore any benefits that fixed-line customers derive from receiving calls from the mobile network.

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<sup>24</sup> Frontier Economics (London) ‘Calculating optimal mark-ups on LRIC for mobile services: A report prepared for Vodafone’ (April 2002), p. 4.

The price  $p_f$  is the sum of origination charges on fixed networks (we assume these are set at costs  $c_f$ ) and a termination charge  $p_t$ . Let the cost of termination on mobiles be  $c_t$ . If  $Q_f(p_f)$  is the total quantity of fixed-to-mobile calls, the network's profits from termination are  $(p_t - c_t)Q_f(p_f)$ .

### **Mobile customers**

The net consumer surplus for mobile callers depends on the price of mobile calls,  $p_m$ , and the price they pay for subscription,  $p_s$ . We assume that the strength of people's demand for calls at price  $p_m$  will vary between individuals. This is denoted by  $\theta$  where  $\theta$  lies in the range  $[0,1]$ . If we assume that all mobile calls terminate on the fixed network<sup>25</sup>, and ignoring any benefits from calls received, the net surplus of a mobile customer of type  $\theta$  can be written as

$$v_m(p_m, \theta) - p_s$$

A consumer will only subscribe if the net surplus is positive. Let  $\theta^*$  denote the critical value of  $\theta$  which would indicate that a customer would subscribe. The value of  $\theta^*$  is given by:

$$v_m(p_m, \theta) - p_s = 0$$

The aggregate net surplus of all mobile customers is:

$$V_m(p_m, p_s) = \int_{\theta^*}^1 [v_m(p_m, \theta) - p_s] g(\theta) d\theta$$

The number of subscribers,  $N$ , will depend on  $p_m$  and  $p_s$  and is given by:

$$N(p_m, p_s) = \int_{\theta^*}^1 g(\theta) d\theta$$

if  $q_m(p_m, \theta)$  is the quantity of calls made by consumer of type- $\theta$ , total mobile calls is given by the expression:

$$Q_m(p_m, p_s) = \int_{\theta^*}^1 q_m(p_m, \theta) g(\theta) d\theta$$

If the marginal cost of mobile calls is  $c_m$ , net profit from mobile calls is  $(p_m - c_m)Q_m(p_m, p_s)$ .

Profit from the mobile operator is the sum of profits from subscription, from mobile calls and from termination, net of fixed and common costs. This is given by the following expression:

$$\pi(p_s, p_m, p_t) = (p_s - k)N(p_m, p_s) + (p_m - c_m)Q_m(p_m, p_s) + (p_t - c_t)Q_f(p_f) - K$$

<sup>25</sup> This is a simplifying assumption which is in keeping with the literature. It is straightforward to extend the model to include mobile-to-mobile calls.

## **Annex 2: Mathematical basis for modelling optimal call termination charges**

where:

$k$  = the marginal cost of having a new subscriber on the network; and

$K$  = the common fixed costs of the company

In this expression,

$(p_s - k)N(p_m, p_s)$  = profits from subscription

$(p_m - c_m)Q_m(p_m, p_s)$  = profits from outgoing calls; and

$(p_t - c_t)Q_f(p_f)$  = profits from terminating fixed to mobile calls.

To find the prices that maximise the sum of consumers' surpluses (for fixed-line and mobile customers) and producers surplus, subject to the break even constraint, the Lagrangian can be written as:

$$L(p_s, p_m, p_t, \lambda) = V_f(N, p_f) + V_m(p_m, p_s) + \pi(p_s, p_m, p_t) + \lambda\pi(p_s, p_m, p_t)$$

The first order conditions for finding the optimal prices are:

$$v(p_f) \frac{\partial N}{\partial p_s} - N + (1 + \lambda) \left[ N + (p_s - k) \frac{\partial N}{\partial p_s} + (p_m - c_m) \frac{\partial Q_m}{\partial p_s} + (p_t - c_t) q_f(p_f) \frac{\partial N}{\partial p_s} \right] = 0$$

$$v(p_f) \frac{\partial N}{\partial p_m} - Q_m + (1 + \lambda) \left[ (p_s - k) \frac{\partial N}{\partial p_m} + (p_m - c_m) \frac{\partial Q_m}{\partial p_s} + Q_m + (p_t - c_t) q_f(p_f) \frac{\partial N}{\partial p_m} \right] = 0$$

$$-Q_f + (1 + \lambda) \left[ (p_t - c_t) \frac{\partial Q_f}{\partial p_t} + Q_m \right] = 0$$

$$\pi(p_s, p_m, p_t) = 0$$

## Annex 2: Mathematical basis for modelling optimal call termination charges

## ANNEX 2.2 MARK-UPS TO RECOVER FIXED AND COMMON COSTS IN ABSENCE OF EXTERNALITIES

Total utility is given by:

$$U(q_1, q_2, \dots, q_n) - C(q_1, q_2, \dots, q_n)$$

At prices  $(p_1, p_2, \dots, p_n)$ , the break-even condition is

$$\sum_{k=1}^n p_k q_k - C(q_1, q_2, \dots, q_n)$$

The Lagrangian for the Ramsey pricing problem is

$$L(q_1, q_2, \dots, q_n, \lambda) = U(q_1, q_2, \dots, q_n) - C(q_1, q_2, \dots, q_n) + \lambda \left[ \sum_{k=1}^n p_k q_k - C(q_1, q_2, \dots, q_n) \right]$$

The first-order condition with respect to  $q_i$  is:

$$\frac{\partial L}{\partial q_i} = \frac{\partial U}{\partial q_i} - \frac{\partial C}{\partial q_i} + \lambda \left[ \frac{\partial}{\partial q_i} \left[ \sum_{k=1}^n p_k q_k \right] - \frac{\partial C}{\partial q_i} \right] = 0$$

where

$$\frac{\partial C}{\partial q_i} \equiv MC_i \equiv \text{the marginal cost of providing service } i;$$

$$\frac{\partial}{\partial q_i} \left[ \sum_{k=1}^n p_k q_k \right] \equiv MR_i \equiv \text{the marginal revenue of good } i; \text{ and}$$

$$\frac{\partial U}{\partial q_i} \equiv \text{the marginal social benefit of an additional unit of good } i$$

Hence the first-order condition for good  $i$  can be rewritten as

$$MB_i - MC_i + \lambda [MR_i - MC_i] = 0$$

rearranging this gives the expression

$$\frac{MB_i - MC_i}{MC_i - MR_i} = \frac{MB_j - MC_j}{MC_j - MR_j}$$

In the absence of externalities, the marginal benefit of good  $i$  ( $MB_i$ ) = the price.

Marginal revenue is given by the expression

$$MR_i = \frac{\partial}{\partial q_i} \left[ \sum_{k=1}^n p_k q_k \right] = p_i + \sum_k q_k \frac{\partial p_k}{\partial q_i}$$

Note that

## Annex 2: Mathematical basis for modelling optimal call termination charges

$$q_k \frac{\partial p_k}{\partial q_i} = p_i \frac{p_k q_k}{p_i q_i} \frac{\partial p_k}{\partial q_i} \frac{q_i}{p_k}$$

$$= p_i r_{ki} \phi_{ki}$$

where  $r_{ki}$  is the relative revenue share between service  $k$  and service  $i$ , and  $\phi_{ki}$  is the elasticity of the inverse demand function (for  $k$ -th price equation with respect to  $i$  quantity).

Using this expression for marginal revenue, we get

$$MR_i = p_i + \sum_k p_i r_{ki} \phi_{ki} = p_i \left[ 1 + \sum_k r_{ki} \phi_{ki} \right]$$

If we represent  $\sum_k r_{ki} \phi_{ki}$  as  $A_i$ , we can rewrite the expression for marginal revenue as  $MR_i = p_i [1 + A_i]$ .

By putting this term into the general Ramsey condition and rearranging terms, we get

$$\frac{p_i - MC_i}{p_i} \frac{1}{A_i} = \frac{p_j - MC_j}{p_j} \frac{1}{A_j}$$

$\frac{1}{A_i}$  is known as the ‘super-elasticity’. Denoting this as  $S_i$ , we can rewrite the expression as

$$\left( \frac{p_i - MC_i}{p_i} \right) S_i = \left( \frac{p_j - MC_j}{p_j} \right) S_j$$

which is analogous to the familiar Ramsey inverse-elasticity rule.

## Annex 2: Mathematical basis for modelling optimal call termination charges

## ANNEX 2.3 OPTIMAL PRICES FOR MOBILE TELEPHONY IN THE PRESENCE OF FIXED AND COMMON COSTS AND NETWORK EXTERNALITIES

We model  $n$  linked demand functions given by

$$q_i = q_i(p_1, p_2, \dots, p_n)$$

These can be inverted to give

$$p_i = p_i(q_1, q_2, \dots, q_n)$$

$U_i$  = the private utility of consuming  $q_i$  of good  $i$  (i.e. the utility gained by the individual who is purchasing it).

External effects are captured by  $q_i$  appearing in the other components of the utility function (i.e.  $U_j$ ).

Hence total welfare is represented by  $U_T$  where

$$U_T = \sum_{i=1}^n U_i$$

Along a good's demand curve, the price equals the marginal private benefit of consuming the good. Therefore

$$\frac{\partial U_i}{\partial q_i} = p_i(q_1, q_2, \dots, q_n)$$

We can therefore derive an expression for total utility,  $U_T$

$$U_T = \sum_{i=1}^n \int p_i(q_1, q_2, \dots, q_n) dq_i$$

The Lagrangian for the optimal quantities is therefore

$$L = U_T - C(q_1, q_2, \dots, q_n) + \lambda \pi(q_1, q_2, \dots, q_n)$$

where  $\pi(q_1, q_2, \dots, q_n)$  = the profit made by the company.

The first-order conditions for this are given by the following  $n$  expressions:

$$\frac{\partial L_i}{\partial q_i} = \frac{\partial U_T}{\partial q_i} - \frac{\partial C}{\partial q_i} + \lambda \frac{\partial \pi}{\partial q_i} = 0$$

$$\text{and } \pi(q_1, q_2, \dots, q_n) = 0$$

Optimal prices can then be derived by putting these quantities into inverse demand equations.

If externalities are included, the standard Ramsey formula can be modified to:

## Annex 2: Mathematical basis for modelling optimal call termination charges



$$\left(\frac{p_j - MC_j}{p_j}\right) S_j = \left(\frac{p_i - MC_i/e}{p_i}\right) \left(\frac{eS_i}{S_i(1-e)+1}\right)$$

We use this as a cross check in the model to make sure that the solution is correct.



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