

Mobile Termination Welfare Analysis

Public Version

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Prepared for

Optus

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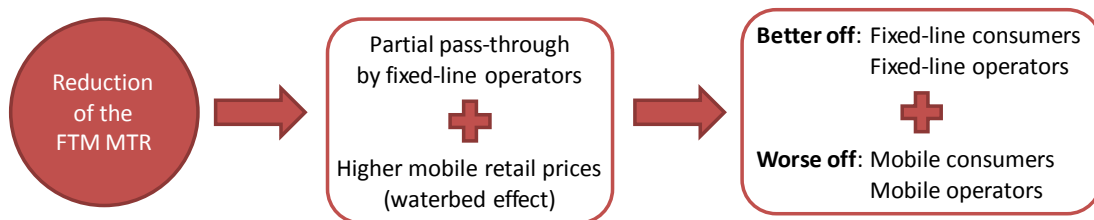
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Executive Summary

This report presents a welfare analysis of reductions in mobile termination rates (MTRs) in Australia below the current level of nine cents per minute. We consider two alternative (factual) scenarios versus a counterfactual of no change. The two factual scenarios both involve a reduction of the MTR to six cents per minute, with one scenario assuming an immediate reduction and the other assuming a two-year glide path is used.

A reduction in the fixed-to-mobile (FTM) MTR will generate welfare benefits in the fixed-line market, and welfare detriments in the mobile market. This is illustrated in Figure 1. The net effect depends on the extent to which MTR reductions are passed through by fixed-line operators, and the corresponding increase in mobile retail prices (the ‘waterbed’ effect).

Figure 1: Effects of changing the FTM MTR.



Using a welfare model calibrated with Australian telecommunications market data, we estimate the net welfare effects of MTR reductions versus the counterfactual over a five year period. We also decompose the total welfare effects into effects on different groups to illustrate the transfers created by changing MTRs. Specifically, we estimate effects on fixed-line consumers, fixed-line operators, mobile consumers, and mobile operators.

We analyse two different models of the fixed-line market. One assumes that lower FTM MTRs are partially passed through to lower FTM per-minute prices. The other assumes that reductions are passed through to lower overall fixed-line ‘bundle’ prices. In both cases, a waterbed effect operates in the mobile retail market and this leads to higher overall mobile prices and reduced mobile subscription.

Key parameters in our analysis are the rate at which lower MTRs are passed through into lower prices in the fixed-line market, and the size of the waterbed effect. Historically, pass-through of MTRs to fixed-line retail prices in Australia has been relatively low, and based on the available evidence we expect that around 20% of MTR reductions will be passed through.

International evidence regarding the size of the waterbed effect suggests that it will be relatively strong but not complete. In our view a conservative estimate of the size of this effect is that mobile operators will effectively recover 50% of lost FTM termination revenues through higher mobile retail prices. This results in mobile retail price increases for Australia that are relatively small in comparison to international estimates of the size of the waterbed effect.

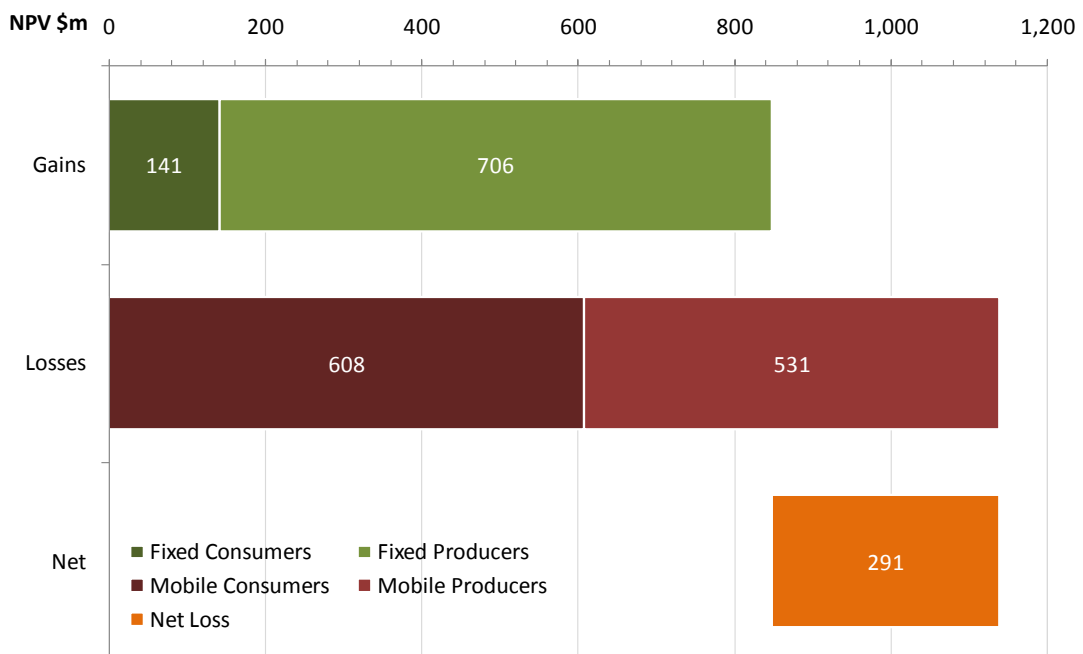
Overall, we estimate that reducing the FTM MTR from nine to six cents per minute will cause FTM per-minute prices to reduce by about 0.6 cents per minute (1.6%), or the price of the average fixed-line bundle will reduce by about \$3.50 per year (0.5%). This will be associated with a waterbed effect in the mobile market corresponding to an increase in the price of the average mobile bundle increasing by about \$6 per year (1.0%).

For all cases, we estimate the change in welfare in the factual scenario relative to the counterfactual. With no glide path, we estimate welfare gains in the fixed-line market of between \$751m and \$847m in NPV terms over five years. Corresponding welfare losses in the mobile market are estimated at between \$1,139m and \$1,148m, leading to an overall welfare loss of between \$291m and \$398m under our baseline assumptions. The negative effects on welfare are somewhat mitigated by the use of a glide path, with net welfare losses of between \$223m and \$306m in that case.

Due to the low rate of pass-through, most of the welfare gains (around 80%) accrue to fixed-line operators, while fixed-line consumers do not benefit greatly from lower FTM MTRs. In the mobile market, the welfare losses due to the waterbed effect are split approximately equally between mobile consumers and mobile operators.

For example, Figure 2 illustrates our estimated welfare results assuming that lower FTM MTRs are passed through into lower FTM per-minute prices and assuming no glide path. With 20% pass-through, fixed-line consumers are better off by \$141m over five years, while fixed-line operators are better off by \$706m due to retained mobile termination cost savings. In the mobile market, higher prices make consumers worse off by \$608m, while mobile operators are unable to recover all of the lost termination revenue and are worse off by \$531m. These relatively large transfers result in a net welfare reduction of \$291m.

Figure 2: Welfare results of the FTM minutes model with no glide path, for baseline parameters.



We also examine the financial implications for mobile operators of changing the mobile-to-mobile (MTM) MTR, using data on existing interconnection traffic balances between the three operators. The net effects on the operators are small in comparison to total mobile retail revenue, and we therefore expect that effects on mobile retail prices will be minimal in comparison with those arising due to the waterbed effect. Changing the MTM MTR will lead to a financial gain for some mobile operators and losses for others, but this is not expected to lead to significant welfare effects in the mobile market.

Finally, we test the sensitivity of our results to the key assumptions. For plausible parameter ranges, the net welfare effects of reducing MTRs are expected to remain negative for the scenarios that we have analysed.

1 Introduction

Mobile termination rates (MTRs) are charged by mobile network operators to other networks for the completion of calls or texts to mobile customers. Under a calling-party-pays regime, MTRs are a way of allowing mobile operators to recover the cost of terminating traffic for which they receive no direct retail revenue.

Two key MTRs are often distinguished – for traffic originating on mobile networks and for traffic originating on fixed networks. In practice the levels of these two MTRs (for voice calls) are often set to be the same, however the two MTRs have different effects due to the different nature of the traffic to which they apply.

In many countries, including Australia, MTRs have been regulated, due to concerns that MTRs set by negotiation between operators may not lead to economically efficient outcomes in retail markets. In practice regulated MTRs have usually been determined by estimating mobile termination costs using a forward-looking cost model, or by benchmarking against MTRs in other countries. A few countries have adopted a bill-and-keep model for mobile termination, however this approach remains very much in the minority compared to the use of cost-oriented MTRs.

For mobile operators, termination is both a source of cost and a source of revenue. The MTR for fixed-to-mobile (FTM) calls is a cost to fixed-line operators and revenue for mobile operators. The MTR for mobile-to-mobile (MTM) traffic is both a cost and revenue for all mobile operators.

This means that a change in the level of MTRs can affect prices and welfare in telecommunications markets in complex ways. It implies that a change in MTRs may make some people better off and others worse off, and create welfare transfers. The sizes of these transfers and the overall change in welfare depend crucially on how telecommunications operators (both fixed and mobile) respond to the change.

Regulators are generally aware of these issues, however the key welfare effects of a change in the MTR are not always quantified. One case where considerable effort has been expended on quantification of the welfare effects of MTR regulation has been in New Zealand. During the New Zealand Commerce Commission's first investigation of mobile termination regulation in 2004, a quantitative cost-benefit model of regulation was developed by the Commission and subsequently refined significantly following submissions from various parties in that investigation.¹ Quantification was again a key feature of the Commission's second mobile termination investigation in 2009, where the 2004 analysis was subject to further refinement and scrutiny.²

We were deeply involved in both of the New Zealand investigations, as expert advisors for Vodafone New Zealand. This included undertaking detailed reviews of the welfare modelling and contributing our own versions of the analysis. In our view the basic structure of the welfare model that has emerged from the New Zealand investigations

¹ See <http://www.comcom.govt.nz/mobile-termination-rates/>.

² See <http://www.comcom.govt.nz/mobiletomobiletermination/>.

captures the key welfare effects flowing from a change in the level of the MTR. We do disagree with the Commerce Commission about some of its inputs and assumptions, and we believe that the Commission's modelling of some variables could be improved in various ways. However, we generally agree with the overall modelling framework that has been used in the New Zealand investigations, and as noted above it has been subject to intense scrutiny from all sides of the regulatory debate.

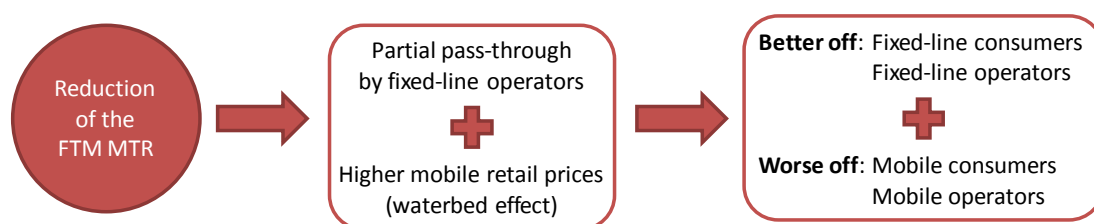
Optus has asked us to undertake a similar welfare analysis for Australia of hypothetical MTR regulatory scenarios, drawing on our experience in the New Zealand investigations. In Australia the MTR for voice calls is currently regulated at nine cents per minute. We have calibrated a welfare model using Australian telecommunications market data, and used this to estimate the welfare effects of different future MTR scenarios relative to a counterfactual of no change. In our results we estimate the overall difference in economic welfare across fixed and mobile telecommunications markets under each scenario relative to the counterfactual. We also present estimates of welfare transfers underlying the net welfare results, to illustrate the gains and losses experienced by different groups.

2 MTR Welfare Modelling Theory

In this section we describe the welfare framework that we have used, and results from the analysis are presented in section 3. For convenience we discuss changes in the FTM MTR and MTM MTR separately, although in our analysis we assume that these two MTRs are set equal to each other, as is usually the case in practice.³ We assume there is a single MTR that applies to all mobile operators, i.e. MTRs are symmetric and reciprocal. We also ignore SMS termination and concentrate on voice.

The key effects of reducing the FTM MTR are illustrated in Figure 3. Fixed-line consumers benefit to the extent that these reductions are passed through by fixed-line operators, and fixed-line operators benefit to the extent that the reductions are not passed through. On the flipside, the waterbed effect (explained below) causes mobile retail prices to rise, making mobile consumers worse off, and mobile operators are also worse off to the extent that the waterbed effect does not fully offset the reduction in FTM termination revenue.

Figure 3: Effects of changing the FTM MTR.



The effects in Figure 3 are relatively simple to model. In contrast, the MTM termination rate is both a source of revenue and cost for mobile operators. In general, traffic flows between mobile networks tend to be relatively balanced, and the net revenues or costs of each operator from MTM termination are usually relatively small in comparison with mobile retail revenues. However, theoretical literature suggests that changing the MTM termination rate can have effects on the incentives of mobile operators and consequently affect mobile retail prices and welfare. We discuss this further in section 2.2 below.

In all cases we are interested in the *difference* in measures of welfare between a factual and a counterfactual scenario. We assume the counterfactual is that the MTR remains unchanged at the current level of nine cents per minute. We consider different factual scenarios representing reductions of the MTR below nine cents per minute.

The latest complete set of relevant data available to us about the Australian telecommunications market covers the year to June 2010. We use this as the starting point for our analysis, and analyse the difference between the factual and counterfactual scenarios over a subsequent five year period. What matters most for our analysis are the

³ There are advantages and disadvantages of having equal FTM and MTM MTRs. Any differences will give rise to incentives for inefficient arbitrage. On the other hand, the socially optimal levels of the FTM and MTM MTRs are not necessarily the same, as they affect mobile competition and prices in different ways (see Armstrong & Wright, 2009).

relative differences between the two scenarios, not the precise path of each scenario over time relative to today. All welfare results are expressed in present value terms, using a social discount rate of 9.5%.⁴

Our results depend on data about the Australian telecommunications market from various public sources, and provided to us by Optus. While we have attempted to verify the data where possible, any errors in the data will affect the reliability of our results.

2.1 Modelling the Effects of Changing the FTM MTR

The FTM MTR is a marginal cost for fixed line operators in the production of FTM calls, and the corresponding payments are revenues for mobile operators. Reducing the MTR will lead to welfare gains in the fixed-line market due to cost reductions. It will also lead to welfare losses in the mobile market as profit-maximising mobile operators respond to lower termination revenues by raising retail mobile prices. This is known as the ‘waterbed’ effect.

2.1.1 Effects in the Fixed-Line Market

The most likely effect of a reduction in the MTR is lower prices for FTM calls, to the extent that fixed operators pass through the cost savings to consumers.

Alternatively, competition among fixed operators could be viewed in terms of competition for a provision of a bundle of services. There may be a single price for the bundle, or a subscription charge together with various usage charges. In that case, the effect of a reduction in the MTR could play out in various ways resulting in an improvement in the overall value of the bundles sold to consumers.

In the bundle case, the link between a change in the FTM MTR and a change in the overall value of the fixed bundle is not simple. The bundle has many dimensions, any of which could be adjusted by fixed operators in response to a change in the MTR. The exact way in which this occurs would depend on detailed characteristics of consumer demand and business strategies that we are not able to observe. In contrast, the link between the MTR and the per-minute price for FTM calls is relatively straightforward, since the MTR is the main component of the marginal cost of these calls.

We have developed two different welfare models to capture these two possibilities:

1. **FTM minutes model:** Reductions in the FTM MTR are passed through to the per-minute price for FTM calls. The relevant market quantity is the volume of FTM minutes per year.

⁴ We are not aware of an appropriate public discount rate for telecommunications cost-benefit analysis in Australia, however the New Zealand Treasury has estimated a social discount rate of 9.5% in telecommunications and related industries – see <http://www.treasury.govt.nz/publications/guidance/planning/costbenefitanalysis>.

2. **Fixed-line bundle model:** Reductions in the FTM MTR are passed through to the overall price of the average fixed-line bundle. The relevant market quantity is the number of fixed-line subscribers.

In our view the FTM minutes model is empirically more robust, because of the indirect link between the MTR and the overall value of fixed-line bundles sold in the bundle model. The following sections describe how we modelled each of these two cases.

FTM Minutes Model

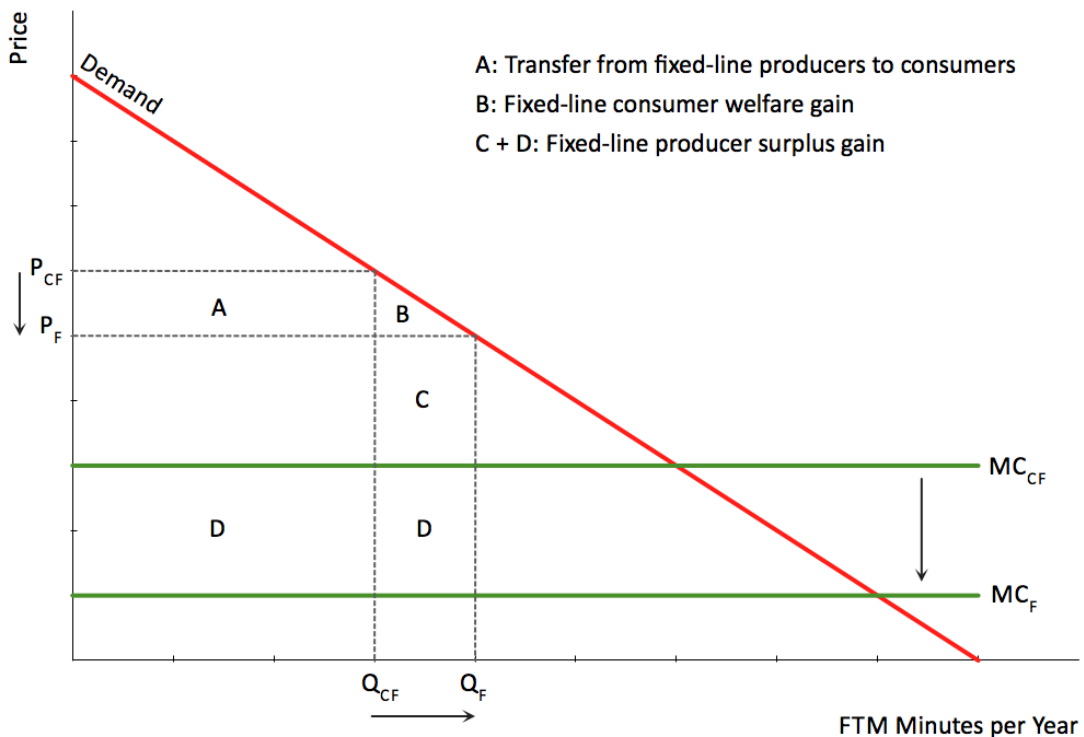
In the counterfactual the MTR does not change, and so we assume FTM prices also do not change over time. In the factual scenario we assume that some percentage of the reduction of the MTR each year is passed through into the FTM retail price. In particular, the FTM retail price in year t , P_t , is given by:

$$P_t = P_{t-1} + \alpha(MTR_t - MTR_{t-1})$$

where α is the rate of pass-through and MTR_t is the MTR in year t .

The FTM retail price in the factual scenario is therefore less than the price in the counterfactual scenario in each year if the MTR falls. The welfare effects of this are illustrated in Figure 4. When the FTM retail price falls from the counterfactual price (P_{CF}) to the factual price (P_F), there is a transfer of welfare from producers to consumers (area A) and a gain in consumer surplus as the quantity increases (area B). Fixed-line firms are worse off by the amount of the transfer (area A), but gain producer surplus due to additional volume (area C) and the fact that a lower MTR reduces their costs (area D).

Figure 4: Welfare effects in the FTM market.



It is important to note that some of the fixed-line producer surplus gain (part of area D) is simply a transfer from mobile operators to fixed operators, due to the lower MTR. This occurs if the MTR reductions are not fully passed through to fixed-line consumers by fixed operators. These are not aggregate welfare gains, but we estimate them in order to illustrate the transfers that occur as a result of regulation.

The per-minute marginal cost in the FTM market faced by fixed-line operators is estimated as the MTR plus one cent per minute for origination costs, plus an additional 10% markup for overhead and retailing costs.

Fixed-line Bundle Model

As discussed above, it is more difficult to estimate the effects of lower MTRs on overall fixed-line bundles because the rate of pass-through is not easily observed or modelled. Attempting to ‘unpack’ fixed-line bundles would also require somewhat arbitrary assumptions about allocation of subscription revenues to services. The profit-maximising configuration of retail prices within the bundle, and pass-through to these prices, depends partly on demand cross-price elasticities that are not easily estimated.

Therefore, in this model we focus on the effect of MTRs on the overall value of the typical bundle consumed in the fixed-line market. We use overall fixed-line average revenue per user (ARPU) as a proxy for the overall ‘price’ that consumers pay for the average bundle of fixed services that they consume. The corresponding ‘quantity’ is the total number of fixed-line subscribers. Consumer surplus in this context can therefore be interpreted as the overall surplus that consumers get from the average bundle of fixed-line services that they use.

Mobile termination is one component of the cost of the overall average fixed-line bundle. We calculate the average annual mobile termination cost per fixed-line subscriber based on the average volume of FTM calls per subscriber, and assume that some percentage of the change in this cost between the factual and counterfactual scenarios is passed through to overall fixed-line prices. Given the resulting price differences between the factual and counterfactual in each year, we estimate welfare effects in the fixed line market in the same fashion illustrated in Figure 4. The only difference is that the price is proxied by fixed-line ARPU and the quantity is the number of fixed-line subscribers.

Marginal cost of the average fixed-line bundle is difficult to observe. We estimate this using operating profit margins. For example, if ARPU is \$500 per year and the operating profit margin is 40%, then we assume marginal cost is \$300. Relative to the counterfactual, in the factual we assume that this marginal cost reduces by the amount of the reduction in average FTM termination cost per fixed-line subscriber.

2.1.2 Effects in the Mobile Market (Waterbed Effect)

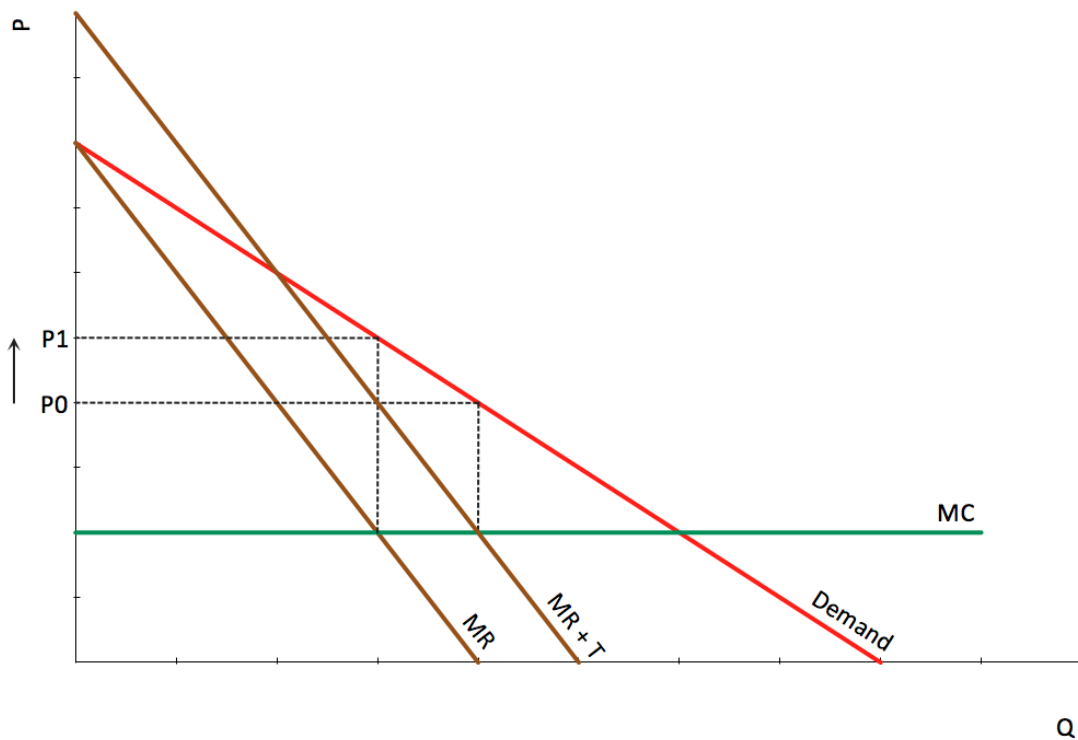
As noted above, lower FTM MTRs make mobile operators worse off. Part of the gains of fixed operators due to reduced costs are simply lost termination revenues of mobile operators. To some extent, this revenue loss will be offset by higher prices in the retail mobile market. This is not because mobile operators try to ‘recover’ the lost revenue, but

rather because their incentives to compete for mobile customers are changed when the MTR changes. The end result is a partial offset of the welfare losses experienced by mobile operators, and additional welfare losses for mobile consumers.

The basic theory of the mobile waterbed effect is illustrated in Figure 5.⁵ This figure shows a stylised representation of the mobile market. The demand curve represents demand for mobile subscriptions as a function of a simple price for subscription. The marginal revenue associated with this demand is shown by the curve labelled MR . This is the marginal retail revenue that mobile operators get directly from their customers. In addition to this retail revenue, each mobile customer also attracts some FTM calls and corresponding FTM termination revenue. If T is the average FTM termination revenue per mobile customer, then total marginal revenue is the curve shown by $MR + T$, which is greater than MR .

Given marginal revenue of $MR + T$, a profit-maximising mobile operator will set a price of P_0 . In contrast, if there is no termination revenue (so that marginal revenue is just MR), a profit-maximising operator will set a price of P_1 . This illustrates how a reduction in the FTM MTR will induce mobile operators to increase retail mobile prices. The waterbed effect arises because of the reduction in marginal revenue per mobile subscriber caused by reducing the MTR. In simple terms, a reduction of the FTM MTR makes mobile consumers less attractive to mobile networks, everything else equal. This causes mobile operators to compete less intensely for customers, and mobile prices rise.

Figure 5: The mobile waterbed effect.



⁵ See Schiff (2008) for a complete discussion.

We model the waterbed effect in slightly different ways for the FTM minutes model and the fixed-line bundle model. In both cases, the objective is to estimate the change in retail mobile prices and the corresponding welfare effects.

Waterbed Effect in the FTM Minutes Model

Although the waterbed effect does not arise through a ‘revenue recovery’ mechanism, it is pragmatic and convenient to model it as such. In the FTM minutes model, we model the waterbed effect by calculating the difference in FTM termination revenues received by mobile operators in the factual scenario versus the counterfactual, in each year. This takes into account both the change in the MTR and the change in FTM minutes as a result of the changes in FTM prices. We then assume that mobile operators ‘recover’ some percentage of the lost termination revenue in the factual relative to the counterfactual in each year through higher mobile retail prices. The proportion of lost revenue recovered represents the strength of the waterbed effect, with 100% recovery corresponding to a ‘full’ waterbed effect.

Waterbed Effect in the Fixed-line Bundle Model

Our approach to modelling the waterbed effect in the fixed-line bundle model is similar in that we assume mobile operators recover some percentage of the FTM termination revenue lost in the factual relative to the counterfactual in each year.

We do not calculate the effects of lower MTRs on FTM per-minute prices explicitly in this model. Instead we focus on overall effects on fixed-line prices. However, the number of fixed-line subscribers changes in the factual relative to the counterfactual, and so does the number of mobile subscribers. These changes will have an impact on the volume of FTM calls even if the per-minute price for FTM does not change.

To model these effects, we assume that the ratio of the number of FTM minutes to the *product* of the number of fixed-line subscribers and number of mobile subscribers remains constant over time. We use this ratio (calculated from 2010 data) to estimate the volume of FTM minutes and hence mobile operators’ FTM termination revenue in the counterfactual and factual in each year, based on the number of fixed-line subscribers and mobile subscribers.

For example, suppose there are 20 fixed-line subscribers and 10 mobile subscribers. There are 200 possible ‘connections’ between these groups of subscribers. Suppose that 100 minutes of calls were made in 2010, then the ratio of minutes to possible connections is 0.5. In future years, the number of fixed-line subscribers changes in the factual relative to the counterfactual as lower FTM MTRs are passed through to fixed-line retail prices. Similarly the number of mobile subscribers changes in the factual relative to the counterfactual as a result of the waterbed effect. We use the ratio (0.5 in this example) to calculate the volume of FTM minutes in the two scenarios in each future year.

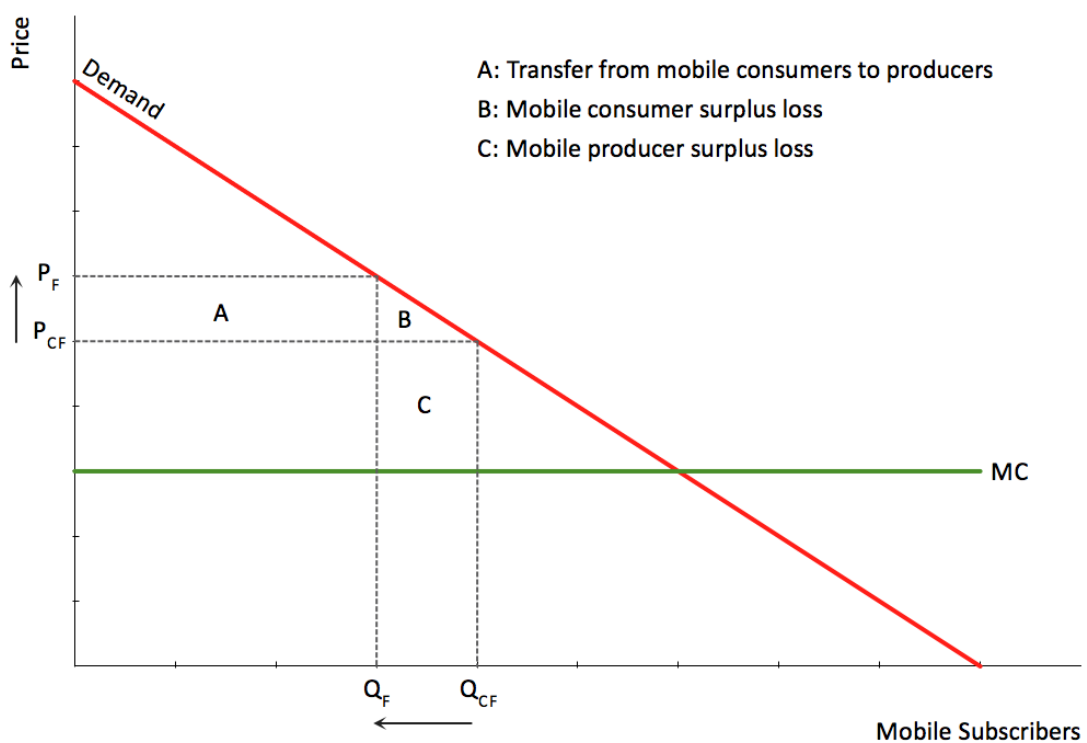
Mobile Market Welfare Effects

We model the overall welfare effects of higher retail prices in the mobile market. Mobile operators charge many different prices, and theory does not clearly predict which of these will rise due to the waterbed effect. Instead, we model the price and demand of the overall average bundle of services that mobile users consume. As described above

for bundles in the fixed market, we use mobile ARPU as an estimate of the total average 'price' of the typical mobile bundle, and the quantity is the number of mobile subscribers. Consumer surplus in this context represents the value that mobile users get from the overall average bundle of services they consume, given the total price paid as estimated by ARPU.

The welfare consequences of the waterbed effect are a transfer from mobile consumers to mobile producers (which partially offsets the producers' loss of FTM termination revenues), and losses of consumer and producer surplus due to reduced mobile subscription. This is illustrated in Figure 6.

Figure 6: Welfare effects in the retail mobile market of the waterbed effect.



The effects shown in Figure 6 are the major mobile market welfare effects resulting from the waterbed effect. Additional second-round welfare effects may also occur following the change in the number of mobile subscribers in the factual relative to the counterfactual. In particular:

- Fewer FTM calls will be made in the factual relative to the counterfactual as there are fewer mobile subscribers who can be called. This will reduce welfare in the fixed-line market.
- Fewer MTM calls will be made in the factual relative to the counterfactual as the remaining mobile subscribers have fewer other mobile users to call. This will reduce welfare in the mobile market.

For reasonable parameter values, the change in the number of mobile subscribers in the factual relative to the counterfactual in each year will be small relative to the total

number of mobile subscribers. For simplicity, we have therefore chosen not to model these additional effects, and our model likely underestimates the negative welfare effects of the waterbed effect.

Finally, to estimate the mobile market welfare effects we need an estimate of mobile operators' marginal cost per mobile subscriber. This cannot be directly observed from data. We therefore estimate marginal cost based on operating profit margins of mobile operators. For example, if operating profit is 40% of revenue then we assume marginal cost is 60% of revenue.

2.2 Effects of Changing the MTM MTR

In practice, the FTM and MTM MTRs are usually set to be the same, and where they are changed by regulation the two MTRs usually move together. This is because of the simple fact that the cost to a mobile network of terminating an FTM call is the same as the cost of terminating an MTM call.

However, changes in the FTM and MTM MTRs have quite different effects on mobile operators as FTM termination is only a source of mobile revenue, whereas MTM termination is a source of both mobile revenues and costs. The theory of the effect of changes in the FTM MTR on mobile operators (i.e. the waterbed effect) is quite well understood and there is empirical evidence to support the existence of this effect.⁶ In contrast, alternative theories of the effects of changing the MTM MTR have been proposed, and there is no clear empirical evidence to prove or disprove these theories.

MTM termination is an example of two-way access pricing. A complete survey of the literature on competition with two-way access is beyond the scope of this report.

However, the following are some examples of contrasting theories:

- Armstrong (1998) and Laffont, Rey & Tirole (1998) showed that with simple linear retail pricing, firms would have an incentive to set two-way access charges above cost, so as to increase each other's effective marginal cost and soften competition. However, this effect disappears if firms compete in two-part tariffs.
- Gans & King (2000) and Armstrong & Wright (2009) showed that, with on-/off-net price discrimination, firms would have an incentive to set two-way access charges below cost, so as to weaken the network effects created by on-/off-net price discrimination and soften competition.
- Harbord & Pagnozzi (2010) cite various papers arguing that below-cost MTRs (or bill and keep) would promote competition in the presence of on-/off-net price discrimination and calling externalities (i.e. uninternalised benefits to call receivers).

All of these papers are theoretical, and we are unaware of any published empirical studies of the effects of changing the MTM termination rate on competition and

⁶ Empirical evidence is given by Genakos & Valletti (2009).

outcomes in mobile markets. Given the complex way that the MTM termination rate affects mobile operators' incentives and profits, and the diversity of pricing and competitive strategies used in practice, it is likely to be difficult to isolate the effects of the MTM termination rate on retail mobile prices through empirical studies.

Since the MTM termination rate affects both the revenues and costs of mobile operators, different forms of this relationship arise depending on the model used. In theory, the MTM MTR will affect incentives to compete for different mobile customer groups, and may affect competitive strategies of mobile operators in different ways, as highlighted by the theoretical literature. The actual effect of changing the MTM MTR will depend on a number of factors, including the preferences of different mobile customer segments, and the customer and traffic profiles of the mobile operators. In contrast, the FTM termination rate is a simple marginal cost for fixed-line operators and a source of per-customer revenue for mobile operators and the theoretical effects on retail prices are much more straightforward.

With these issues in mind, one way to get a sense of the effects of changing the MTM MTR on mobile retail prices is to examine the MTM traffic balances between networks and estimate the financial implications for the operators of a change in the MTR. In general, MTM traffic flows between networks are relatively balanced, and net interconnection payments are relatively small as a result. However, depending on pricing strategies and the mix of customers that each network attracts, some MTM traffic imbalances can persist, meaning that some networks may earn net interconnection revenue from MTM, while MTM is a net cost for other networks.

We have estimated the financial effects of a reduction in the MTM termination rate on each of the three mobile networks in Australia, using data on actual MTM traffic flows. Overall these financial effects are small relative to total mobile revenues and costs, and represent both gains and losses. We therefore do not expect significant effects on overall retail mobile prices due to financial impacts, although there may be a small rebalancing of prices between operators. This analysis is discussed in section 3.4 below.

With the data available to us at this time, an assessment of the broader strategic implications of changing the MTM MTR in Australia is outside the scope of this report. However as noted above, in our view the theoretical predictions about such effects are unclear, and that makes any such analysis difficult.

Given the small financial effects of changing the MTM MTR in Australia, and the lack of clear theoretical or empirical evidence about how this will affect mobile retail prices, we have assumed that there will not be significant welfare effects as a result. At least, it seems likely that these welfare effects will be significantly smaller than the welfare effects in the fixed market and the mobile market arising from changing the FTM MTR. Accordingly, our quantitative analysis below concentrates on the FTM MTR.

3 Model Calibration and Results

This section presents our quantitative results. We discuss the factual and counterfactual scenarios that we have used, and explain how we calibrated the other parameters of the model. As explained above, the relationships between the FTM MTR and retail prices are more straightforward, so we first present welfare effects for a change of the FTM MTR in isolation. We then consider financial effects of changing the MTM MTR. We conclude by testing the sensitivity of our results to some key assumptions.

3.1 Factual and Counterfactual MTR Scenarios

Both scenarios take as the starting point the current regulated MTR of nine cents per minute, and other market data for the year ended June 2010. The counterfactual assumes that the MTR remains constant at nine cents per minute for the following five years. As a result, retail prices and market quantities in the counterfactual are unchanged from their initial values over the five years.⁷

We consider two alternative factual scenarios regarding the MTR. These are intended to reflect an estimate of what a new regulated MTR might be for Australia for the purposes of this modelling exercise, rather than an accurate estimate of the actual cost of mobile termination in Australia.

Our factual scenario is based on the mobile termination cost modelling undertaken by WIK for the ACCC. The most recent update to this model arrived at a cost of 5.9 to 6.2 cents per minute (depending on market share).⁸ However, the ACCC determined that this was a “lower bound” estimate of the cost of an efficient operator in Australia, and consequently regulated termination at nine cents per minute. Based on this, we assume the factual scenario involves reductions of MTRs to six cents per minute. This represents approximately a 33% reduction compared to the counterfactual.

It is common for regulators to use glide paths to smooth the transition to new MTRs. We therefore consider two alternative factual scenarios – one with a glide path and one without. We have benchmarked MTR glide paths used by regulators around the world, and found that the median duration is slightly over two and a half years.⁹ Our welfare model is annual, so for simplicity we assume a linear two-year glide path, noting that

⁷ As noted above, we are only interested in forecasting the *relative* difference between the factual and counterfactual scenarios over time.

⁸ *Domestic Mobile Terminating Access Service Pricing Principles Determination and indicative prices for the period 1 January 2009 to 31 December 2011*, ACCC, March 2009.

⁹ *Mobile Termination Glide Path Benchmarking*, Covec report for Vodafone New Zealand, 7 February 2011, available at <http://www.comcom.govt.nz/assets/Telecommunications/STD/MTAS/Submissions-on-draft-MTAS-STD/Covec-report-on-Glidepath-Benchmarking-for-Vodafone-submission-on-draft-MTAS-STD-7-February-2011.PDF>. A subsequent update is also available at <http://www.comcom.govt.nz/assets/Telecommunications/STD/MTAS/Pre-conference-info-requests/Attachment-to-Vodafone-letter-providing-additional-information-for-MTAS-STD-process-2-March-2011-Covec-report-on-updated-MTAS-glide-path-benchmarking.PDF>.

this is a little shorter than is typically applied in practice. Table 1 summarises the MTRs used in our counterfactual and factual scenarios.

Table 1: Factual and counterfactual MTR scenarios (cents per minute).

Scenario	Year 1	Year 2	Year 3	Year 4	Year 5
Counterfactual	9.0	9.0	9.0	9.0	9.0
Factual 1 (no glide path)	6.0	6.0	6.0	6.0	6.0
Factual 2 (glide path)	8.0	7.0	6.0	6.0	6.0

3.2 Calibration of Model Parameters

In addition to the MTR scenarios above, our model relies on a number of other parameters in order to calibrate demands and costs, and generate welfare estimates. For simplicity we assume linear demand, noting that linear demand approximates other demand curves for relatively small price changes.¹⁰ The Appendix shows how the two parameters of a linear demand curve can be calibrated from market data on price, quantity, and an elasticity estimate.

Table 2 shows the baseline parameter values that we used and our information sources. All model parameters were calibrated using Australian telecommunications market data from public sources and provided to us by Optus. Most data relate to the year ended June 2010. In section 3.5 we test the sensitivity of our results to changes in the key parameters for which there is uncertainty, such as demand elasticities and pass-through.

Additional details of how we calibrated the fixed-line pass-through, waterbed effect, and demand elasticity parameters are given below.

¹⁰ We also tested a constant-elasticity demand specification, however this had a minimal effect on our results as the estimated price changes are relatively small, and hence linear demand closely approximates other demand functions such as constant elasticity.

Table 2 Calibration of baseline model parameters.

Parameter	Value	Sources and Notes
Fixed-line Market		
FTM Minutes Model		
FTM price	37.38 cents per minute	Average of business and residential for Telstra, calculated as retail revenue divided by volume. ¹¹
FTM quantity	[c-i-c] million minutes per year	Covec estimate. ¹²
Marginal cost per minute	FTM MTR + 1 cent per minute + 10%	1 cent per minute for origination costs, ¹³ and estimated additional 10% markup for overhead ¹⁴
Demand elasticity	-0.60	Estimate used by NZ Commerce Commission ¹⁵ and Analsys Mason ¹⁶
Pass-through to retail prices	20%	See below
Fixed-line Bundle Model		
Bundle price	\$656 per year	Covec estimate (weighted average) ¹⁷
Number of subscribers	10.37 million	Covec estimate. ¹⁸
Marginal cost per subscriber	\$302 per year	Estimated based on industry profit margins.
Demand elasticity	-0.60	Same as FTM minutes model
Pass-through to retail prices	20%	See below
Mobile Market		
Mobile termination cost	6 cents per minute	See section 3.1 above
Bundle price	\$604 per year	Covec estimate (weighted average) ¹⁹
Number of subscribers	26.73 million	Sum of Telstra, Optus and VHA subscribers ²⁰
Mobile operators' marginal cost per subscriber	50% of revenue	Estimated based on industry profit margins.
Demand elasticity	-0.43	Estimate used by NZ Commerce Commission ²¹
Waterbed effect	50%	Based on Genakos & Valletti (2009)

¹¹ ACCC imputation testing reports. <http://www.accc.gov.au/content/index.phtml/itemId/670198>

¹² Telstra FTM minutes obtained from ACCC imputation testing reports. Optus FTM minutes provided by Optus. Other FTM minutes estimated from 2008/09 market shares from ACCC telecommunications report 2008/09, June 2010.

¹³ ACCC imputation testing reports, as above.

¹⁴ Based on the 10% markup for common organisational-level costs used in the WIK termination cost model for Australia; see *MTAS Pricing Principles Determination 1 July 2007 to 31 December 2008*, ACCC, November 2007.

¹⁵ NZ Commerce Commission, Draft Report on whether the mobile termination access services should become designated or specified services, June 2009.

¹⁶ Anaysys Mason Report for the ACCC, Regulatory treatment of fixed-to-mobile passthrough, October 2009. <http://www.accc.gov.au/content/index.phtml/itemId/848783>

¹⁷ Telstra ARPU from Telstra annual report 2010. Optus ARPU from SingTel annual reports, available at <http://info.singtel.com/about-us/investor-relations/annual-reports>.

¹⁸ Telstra subscribers from Telstra annual report 2010. Optus subscribers provided by Optus. Other subscribers estimated from market shares from ACCC telecommunications report 2008/09, June 2010.

¹⁹ Telstra from Telstra annual report 2010. Optus ARPU from SingTel annual reports, available at <http://info.singtel.com/about-us/investor-relations/annual-reports>. VHA from Hutchinson half year presentation, June 2010.

²⁰ See footnote 19.

²¹ See footnote 15.

3.2.1 Fixed-line Pass-through

In the baseline case we assume that 20% of reductions in the FTM MTR will be passed through to fixed-line prices (either FTM per-minute prices, or the overall price of the fixed-line bundle). This is based on the following available evidence about fixed-line pass-through:

- A 2009 study for the ACCC by Analysys Mason found pass-through in Australia of around 50% in 2004/05 falling to around 25% in 2007/08.²²
- The same Analysys Mason study found average pass-through across ten European countries of around 50%, and this included some countries where pass-through obligations are regulated.
- Using data on Telstra's average FTM prices and MTRs in Australia, we estimated average pass-through of around 18% between 2004 and 2010.
- The EC found that pass-through in Europe may be as low as 20%.²³
- Using econometric analysis we have estimated historic FTM pass-through in New Zealand of around 40%.²⁴

This evidence indicates relatively low levels of pass-through in the fixed-line market in Australia. In our view, 20% is a reasonable estimate of the likely rate of pass-through of further MTR reductions to fixed-line prices in Australia. Below we test the sensitivity of our results to variations of this parameter between 0% and 50%.

3.2.2 Waterbed Effect

The best evidence about the strength of the waterbed effect comes from the empirical study by Genakos & Valletti (2009). Using a panel dataset of twenty countries over six years, they estimated that given the average 10% reduction in MTRs across these countries, the result of the waterbed effect was an increase of mobile retail prices of between 2% and 15%, with an average of 5%. They concluded that the strength of the waterbed effect was "high" but "not complete", i.e. mobile operators do not pass through all of the cost increases that they face to customers, as expected in a situation of imperfect competition.

Accordingly, in our baseline case we assume that mobile operators respond to lower FTM MTRs by 'recovering' 50% of the lost termination revenues through higher mobile retail prices. As will be shown below, this results in an estimate of around a 1% increase in mobile retail prices for Australia, following the 33% reduction in MTRs. This is a

²² *Regulatory treatment of fixed-to-mobile passthrough*. Analysys Mason, October 2009.

²³ *Commission Staff Working Document accompanying the Commission Recommendation on the Treatment of Fixed and Mobile Termination Rates in the EU: Implications for Industry, Competition and Consumers*, 7 May 2009; SEC (2009) 599, page 20.

²⁴ *MTAS Regulation Quantitative Analysis*, Covec, 27 July 2009.

conservative estimate relative to the Genakos & Valletti results, and we also test the sensitivity of our results to recovery rates between 25% and 75%.

3.2.3 Demand Elasticities

Our demand elasticity estimate of -0.60 for the fixed-line market is based on the elasticity estimates surveyed by the ACCC in its 2004 Mobile Services Review.²⁵ The ACCC found a range of elasticity estimates between -0.40 and -0.80, and used the mid-point of -0.60 in its analysis. The same value was also used by the New Zealand Commerce Commission in its mobile termination welfare model.²⁶

Our mobile demand elasticity of -0.43 is an average of eight estimates from different econometric studies surveyed by a 2004 submission by CRA to the ACCC.²⁷ The New Zealand Commerce Commission also used the same elasticity estimate in its mobile termination welfare model.

3.3 Estimated Welfare Effects of Changing the FTM MTR

Using the framework described in section 2, the prices, quantities and welfare estimates predicted by the FTM minutes and fixed-line bundle models are shown in the tables and figures below, for the baseline parameter values in Table 2. Table 3 shows the prices and quantities of the FTM minutes model in the different scenarios, and Table 4 shows the same for the fixed-line bundle model.

Table 3: Prices and quantities – FTM minutes model (June years).

	2010	2011	2012	2013	2014	2015	2016
FTM price (\$ per minute)							
Counterfactual	0.3738	0.3738	0.3738	0.3738	0.3738	0.3738	0.3738
Factual (No Glide)		0.3738	0.3678	0.3678	0.3678	0.3678	0.3678
Factual (Glide)		0.3738	0.3718	0.3698	0.3678	0.3678	0.3678
FTM quantity (m minutes)							
Counterfactual	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]
Factual (No Glide)		[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]
Factual (Glide)		[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]	[c-i-c]
Mobile price (\$ per year)							
Counterfactual	604	604	604	604	604	604	604
Factual (No Glide)		604	610	610	610	610	610
Factual (Glide)		604	606	608	610	610	610
Mobile subscribers (m)							
Counterfactual	26.73	26.73	26.73	26.73	26.73	26.73	26.73
Factual (No Glide)		26.73	26.62	26.62	26.62	26.62	26.62
Factual (Glide)		26.73	26.69	26.66	26.62	26.62	26.62

²⁵ *Mobile Services Review: Mobile Terminating Access Service*, ACCC, June 2004. See page 154.

²⁶ *Schedule 3 Investigation into Regulation of Mobile Termination: Final Report*. New Zealand Commerce Commission, 9 June 2005. See page 101 and 121.

²⁷ *Pricing Mobile Termination in Australia*, Charles River Associates, 22 December 2004. See page 35.

Table 4: Prices and quantities – fixed-line bundle model (June years).

	2010	2011	2012	2013	2014	2015	2016
Fixed bundle price (\$ per year)							
Counterfactual	656	656	656	656	656	656	656
Factual (No Glide)		656	653	653	653	653	653
Factual (Glide)		656	655	654	653	653	653
Fixed subscribers (m)							
Counterfactual	10.37	10.37	10.37	10.37	10.37	10.37	10.37
Factual (No Glide)		10.37	10.40	10.40	10.40	10.40	10.40
Factual (Glide)		10.37	10.38	10.39	10.40	10.40	10.40
Mobile price (\$ per year)							
Counterfactual	604	604	604	604	604	604	604
Factual (No Glide)		604	610	610	610	610	610
Factual (Glide)		604	606	608	610	610	610
Mobile subscribers (m)							
Counterfactual	26.73	26.73	26.73	26.73	26.73	26.73	26.73
Factual (No Glide)		26.73	26.62	26.62	26.62	26.62	26.62
Factual (Glide)		26.73	26.69	26.65	26.62	26.62	26.62

In the FTM minutes model, given our assumptions about fixed-line pass-through, a reduction of the MTR from nine cents per minute to six cents per minute (a 33% reduction) leads to an overall reduction in the FTM price per minute of about 0.6 cents per minute (1.6%) in the factual relative to the counterfactual. This leads to an increase in FTM minutes of about [c-i-c] million minutes per annum (1.0%). The corresponding waterbed effect is an increase in mobile prices of around \$6 per year (1.0%), leading to a reduction of around 115,000 mobile subscribers (0.4%).

In the fixed bundle model, the average fixed bundle price reduces by about \$3.50 per year (0.5%) in the factual relative to the counterfactual, and the number of fixed-line subscribers increases by about 33,000 (0.3%). The corresponding waterbed effect is very similar to the FTM minutes model – an increase of around \$6 per year and a reduction of 115,000 mobile subscribers.

The differences between the counterfactual and factual prices and quantities result in the changes in welfare as illustrated in the following figures, using our baseline parameter values. The welfare changes are expressed in present value terms over the five year period.

Figure 7 shows the differences in welfare between the factual and counterfactual scenario using the FTM minutes model and assuming no glide path. This results in welfare gains in the fixed-line market of \$141m and \$706m for consumers and producers respectively. This is outweighed by losses in the mobile market of \$608m for consumers and \$531m for producers. The result is a net loss of welfare of \$291m.

Figure 7: Welfare results of the FTM minutes model with no glide path, for baseline parameters.

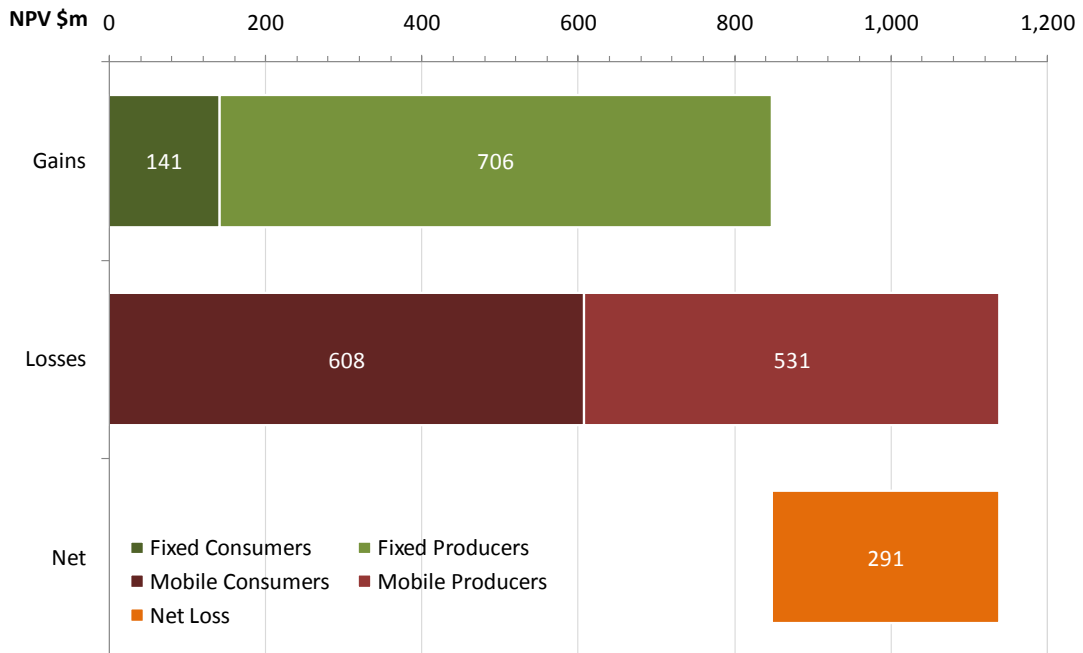


Figure 8 shows welfare results from the FTM minutes model assuming a glide path in the factual scenario. This results in welfare gains in the fixed-line market of \$109m and \$542m for consumers and producers respectively. This is outweighed by losses in the mobile market of \$467m and \$408m. The result is a net loss of welfare of \$223m. Including a glide path in the model has reduced the negative impact of lower MTRs on welfare, as expected.

Figure 8: Welfare results of the FTM minutes model with glide path, for baseline parameters.

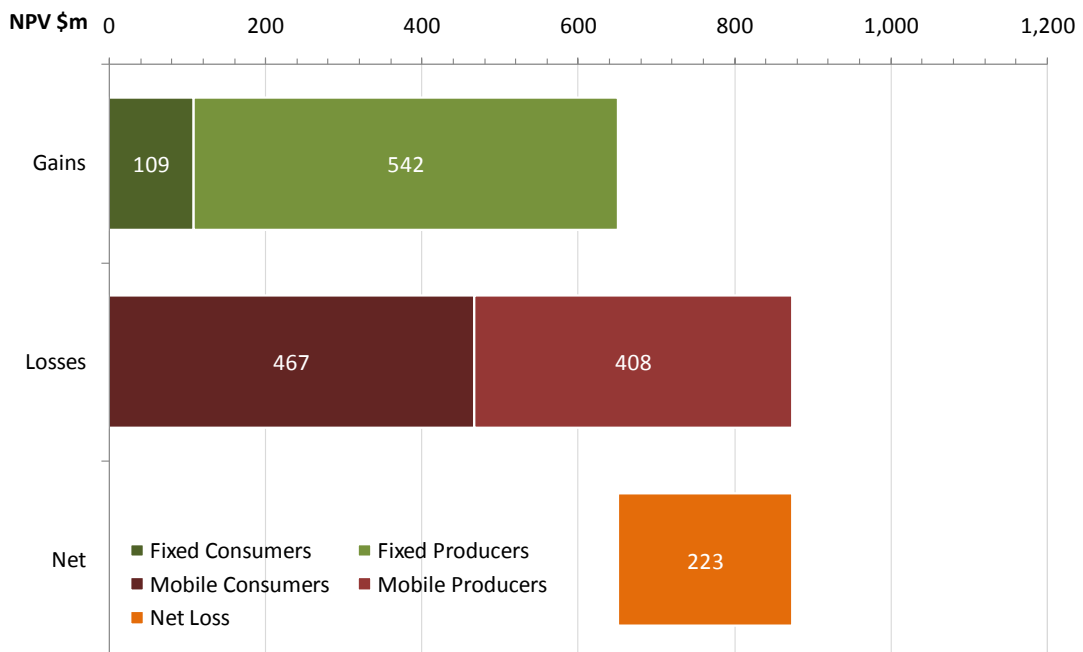
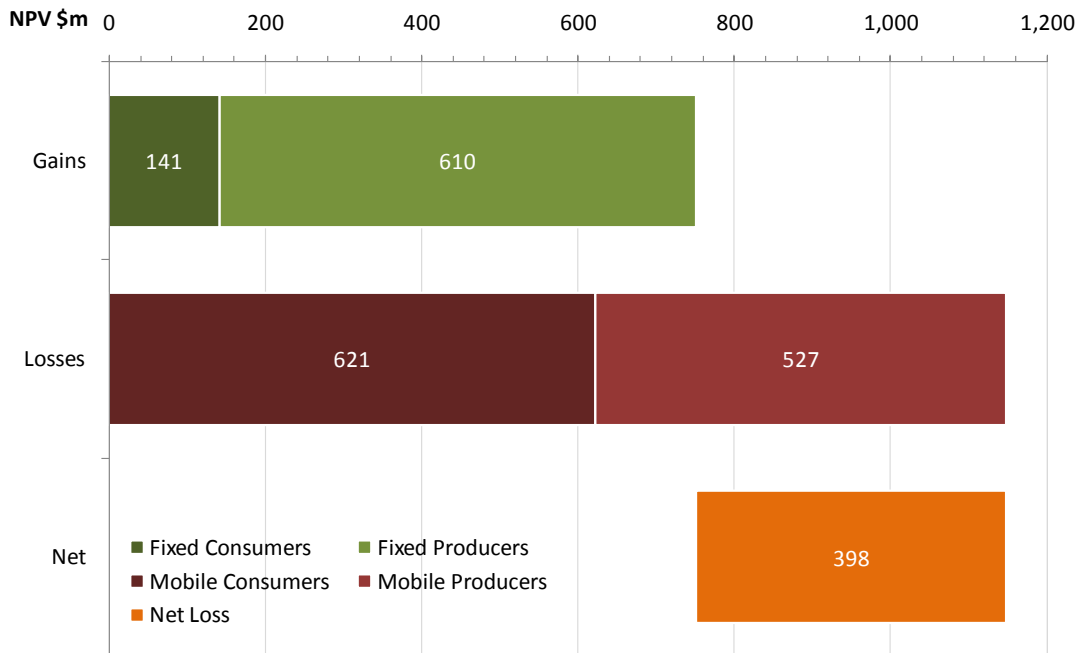


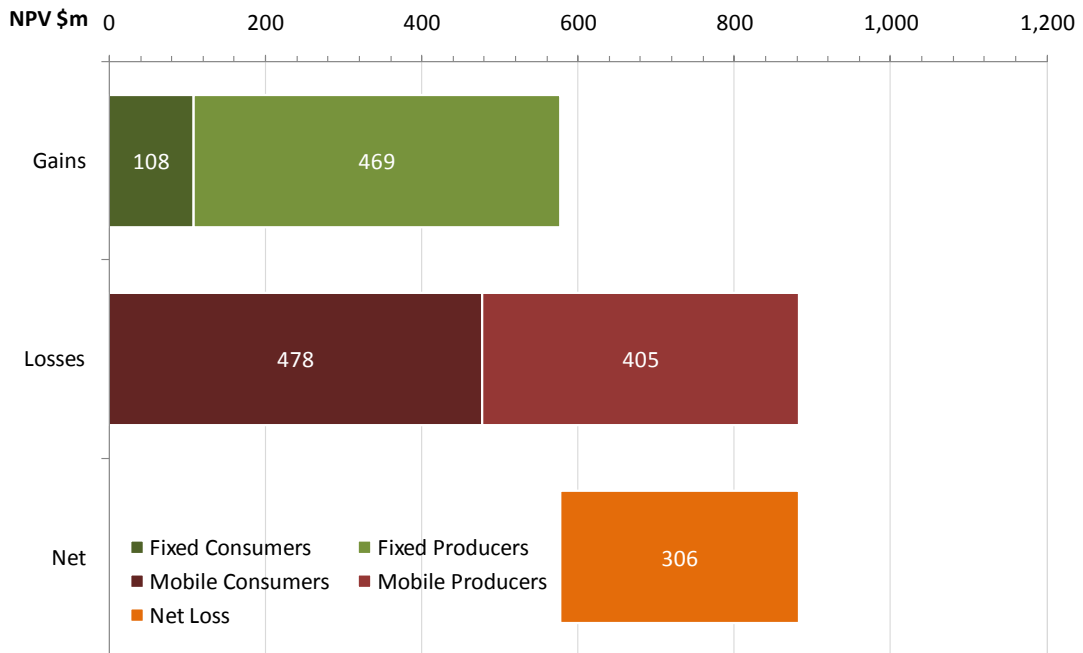
Figure 9 shows welfare results from the fixed-line bundle model with no glide path. This results in welfare gains in the fixed-line market of \$141m and \$610 m for consumers and producers respectively. This is outweighed by losses in the mobile market of \$621m and \$527m. The result is a net loss of welfare of \$398m.

Figure 9: Welfare results of the fixed-line bundle model with no glide path, for baseline parameters.



Finally, Figure 10 shows welfare results from the fixed-line bundle model with a glide path in the factual scenario. This results in welfare gains in the fixed-line market of \$108m and \$469m for consumers and producers respectively. This is outweighed by losses in the mobile market of \$478m and \$405m. The result is a net loss of welfare of \$306m. Including a glide path in the model has again lowered the impact on welfare.

Figure 10: Welfare results of the fixed-line bundle model with glide path, for baseline parameters.



The FTM minutes model and fixed-line bundle model produce similar welfare results in both the fixed and mobile markets for consumers. However, the fixed-line bundle model predicts a somewhat smaller gain for producers in the fixed-line market, and this results in a larger net welfare loss in this model relative to the FTM minutes model.

The reason for the difference is largely due to the different margin on FTM minutes versus the fixed-line bundle implicit in our model. In particular, with an MTR of nine cents per minute, we estimate the cost to fixed-line operators of FTM calls as 11.1 cents per minute, versus a price of around 37 cents per minute, giving a margin of around 70% of revenue. In contrast, we estimate the overall margin on the average fixed-line bundle as 54% of revenue. The greater margin on FTM minutes means that fixed-line operators benefit more when the volume of FTM minutes increases in the factual relative to the counterfactual scenario.

3.4 Estimated Effects of Changing the MTM MTR

As discussed above, we have estimated the financial effects on the three mobile operators of changing the MTM MTR based on existing net voice traffic flows. Optus provided us with confidential data about traffic flows between itself and Telstra and VHA. However, we do not have any information about traffic flows between Telstra and VHA. Based on the data provided to us, there does not appear to be a clear relationship between market shares and net traffic flows. This may be due to the different customer profiles of the operators, and/or different pricing strategies. We

therefore estimated the balance of MTM voice traffic between Telstra and VHA based on the balance of traffic between Telstra and Optus.²⁸

Using this data, we estimated the financial effects on the three mobile operators of a change in the MTM MTR. Some operators are financially better off while others are made worse off. However, the financial effects are small in comparison with total mobile retail revenues. Even if we assume that mobile operators pass through 100% of these effects to mobile retail prices, we estimate that the changes in prices will be trivial. As a result, some consumers will be better off and others will be worse off, but the overall effects on mobile consumers will be minor relative to the waterbed effects induced by changing the FTM MTR.

3.5 Sensitivity Testing

The majority of the parameters used in calibrating our welfare model have been obtained from actual Australian market data. However, there is some uncertainty associated with pass-through rates (both in the fixed-line market, and the size of the waterbed effect), and for the demand elasticity estimates. We have therefore tested the sensitivity of our overall welfare results to changes in these parameters. We did this by generating results from the model for all values of each of these parameters within a reasonable range around its baseline value. Table 5 gives the key parameters along with the base, high and low values used in sensitivity testing.

Table 5: Ranges of values of the key parameters used in sensitivity testing.

Parameters	Base	Low	High	Increment
FTM minutes pass through	20%	0%	50%	5%
Fixed bundle pass through	20%	0%	50%	5%
Waterbed rate	50%	25%	75%	5%
FTM demand elasticity	-0.60	-0.30	-0.80	-0.05
Fixed subscription elasticity	-0.60	-0.30	-0.80	-0.05
Mobile subscription elasticity	-0.43	-0.20	-0.70	-0.05

The models were rerun varying each parameter value separately within the specified range. The results in terms of total net welfare effects are shown in Figure 11 to Figure 14 below. The large red dot shows the NPV of the net welfare in the baseline case and the other dots are the net welfare for different values of the relevant parameter, while keeping all other parameters at their baseline values.

The results are most sensitive to mobile subscription elasticity. An elasticity value of -0.7 results in net welfare losses of \$879m and \$1,021m for the FTM minutes and fixed-line bundle models with no glide paths respectively, whereas an elasticity value of -0.2 results in net welfare losses of \$115m and \$216m. This sensitivity reflects the fact that detriments due to the waterbed effect can be relatively large if mobile subscriptions are

²⁸ In particular, we assume the balance of traffic between Telstra and VHA is the same as between Telstra and Optus, with a small adjustment to account for the different mobile market shares of VHA and Optus.

relatively elastic and the resulting change in mobile prices leads to a larger change in the number of mobile subscribers.

Similarly, the results are also quite sensitive to the waterbed recovery rate. The difference between using a waterbed rate of 25% and 75% is approximately \$450m in net welfare for the models without a glide path and about \$350m with a glide path. Results fairly robust in regards to the choice of fixed pass through rates and fixed elasticities.

In all cases tested in the sensitivity analysis, the net welfare result remains negative.

Figure 11: Sensitivity analysis of the net welfare change in the FTM minutes model with no glide path (NPV \$m).

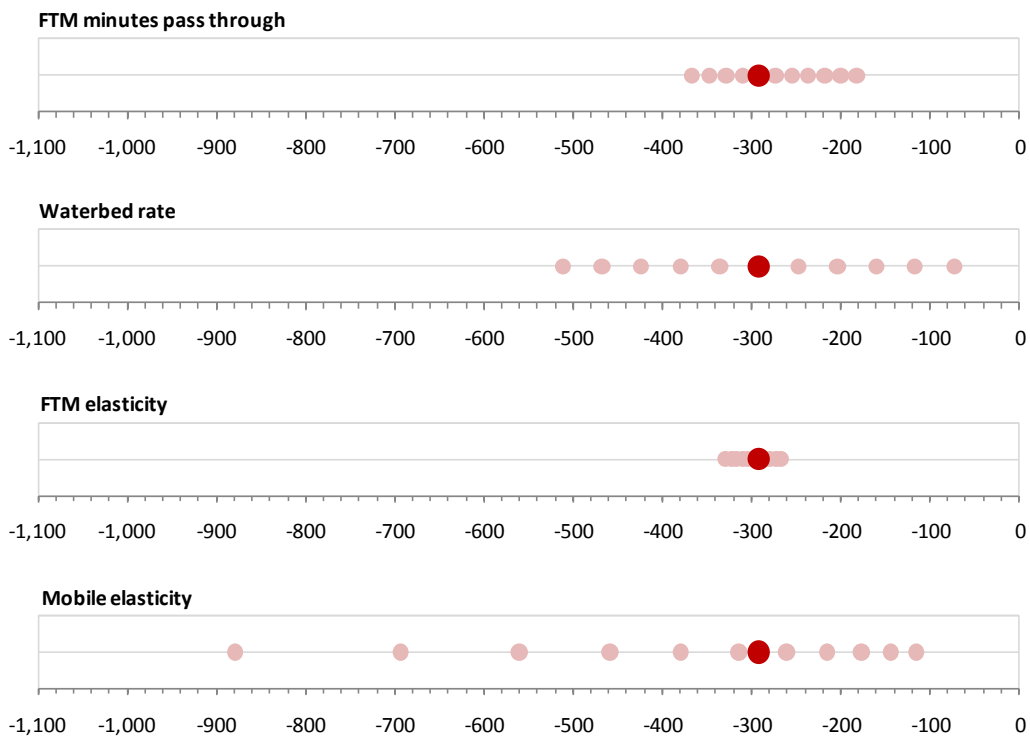


Figure 12: Sensitivity analysis of the net welfare change in the FTM minutes model with glide path (NPV \$m).

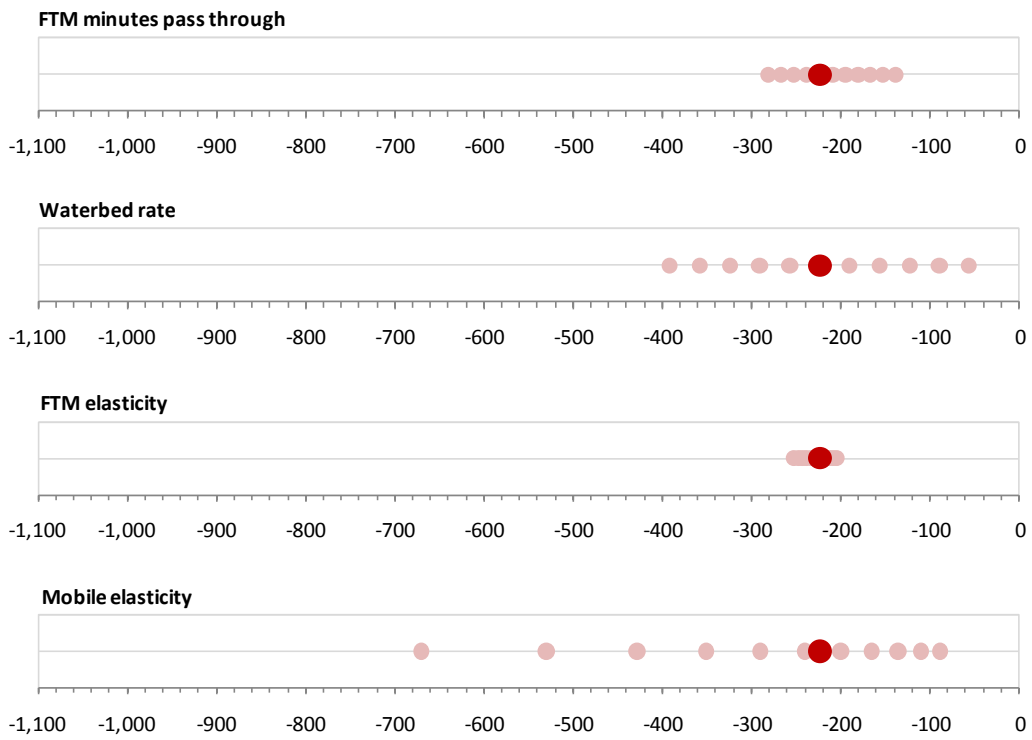


Figure 13: Sensitivity analysis of the net welfare change in the fixed-line bundle model with no glide path (NPV \$m).

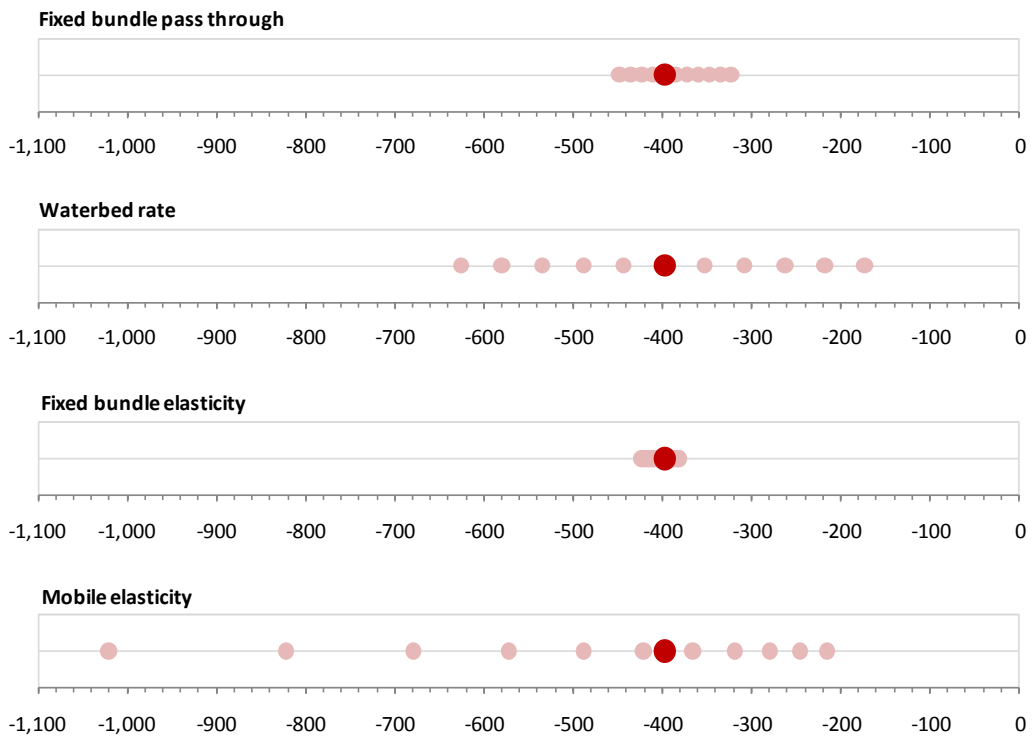
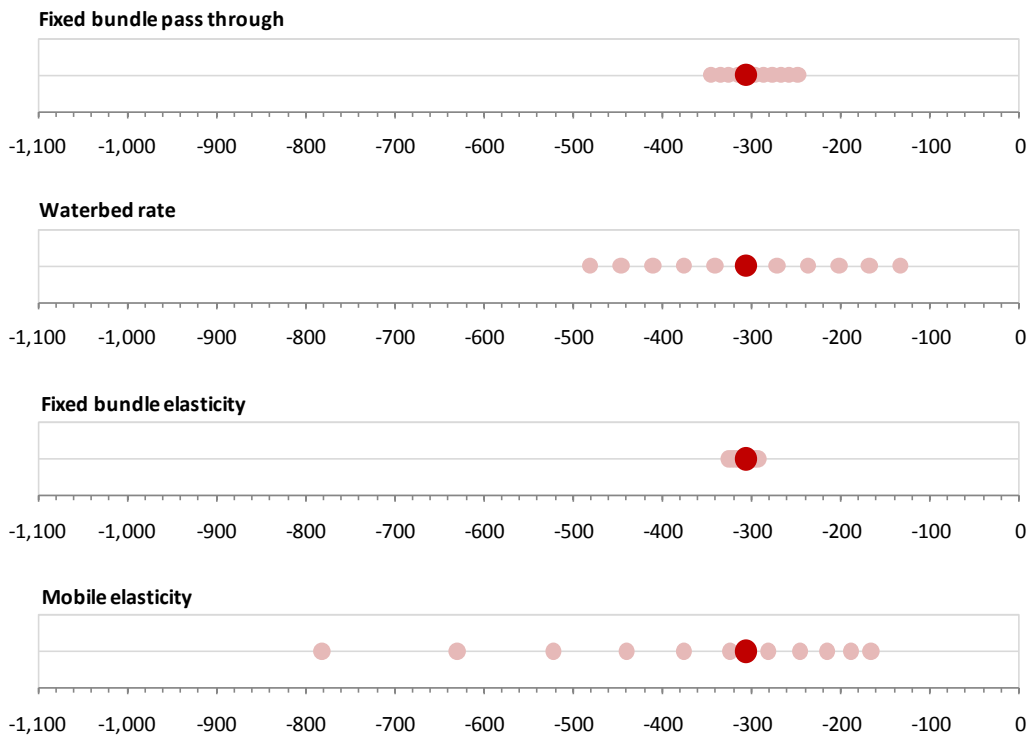


Figure 14: Sensitivity analysis of the net welfare change in the fixed-line bundle model with glide path (NPV \$m).



3.6 Summary and Conclusions

Our analysis above has highlighted the key welfare effects of changing the MTR in terms of gains in the fixed-line market due to lower prices and losses in the mobile market due to the waterbed effect. In the scenarios we analysed, the net overall effect on welfare is negative. This is partly due to relatively low pass-through in the fixed-line market.

Another important conclusion from our analysis is that regulation creates large welfare transfers that underlie the net effect. Profits are transferred from mobile operators to fixed operators to the extent that lower MTRs are not passed through to fixed-line retail prices. Similarly, the waterbed effect leads to transfers away from mobile consumers and towards mobile producers. In general these transfers are significantly larger than the net welfare effects that are created. Notably, if pass-through in the fixed-line market is low then regulation creates large transfers from mobile operators and consumers to fixed operators, and could potentially affect the balance of competition between fixed and mobile operators

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Appendix: Calibrating Linear Demand

Assuming linear demand of the form $Q = a - bP$, the two demand parameters a and b can be calibrated from a single price-quantity point and knowledge of demand elasticity at that point.

For linear demand, price elasticity is given by:

$$\varepsilon = \frac{dQ}{dP} \times \frac{P}{Q} = -b \frac{P}{Q}.$$

Solving the two equations $Q = a - bP$ and $\varepsilon = -bP/Q$ simultaneously for a and b gives:

$$a = Q(1 - \varepsilon)$$

and

$$b = -\frac{\varepsilon Q}{P}.$$